



CIS1910 Discrete Structures in Computing (I)
 Winter 2019, Assignment 1

All answers must be justified in a clear, concise and complete manner. If two answers require the same explanations, justify your first answer only, and refer the reader to that justification for the second answer.

PART A (2+2+1+2+2+3+1+3=16 marks)

1. Find a pair set S such that:

- (a) S does not contain 0 and does not contain $\{0\}$ either.
- (b) S contains 0 but does not contain $\{0\}$.
- (c) S does not contain 0 but contains $\{0\}$.
- (d) S contains both 0 and $\{0\}$.

2. Find a pair set S such that:

- (a) $\{1\}$ does not belong to S and $\{1\}$ is not a subset of S .
- (b) $\{1\}$ belongs to S and $\{1\}$ is not a subset of S .
- (c) $\{1\}$ does not belong to S and $\{1\}$ is a subset of S .
- (d) $\{1\}$ belongs to S and $\{1\}$ is a subset of S .

3. Consider three sets A , B and C such that: each one of these sets contains an element that does not belong to any of the other two sets; the three sets do not have any element in common; any two of them have at least one element in common. Illustrate these relationships with a Venn diagram.

- 4. (a) Find the set of all the subsets of $\{\}$.
- (b) Find the set of all the subsets of $\{\{\}\}$.
- (c) Find the set of all the subsets of $\{\{\}, 0\}$.
- (d) Find the set of all the subsets of $\{\{\}, 0, 1\}$.

5. (a) What are the sets, if any, that do not have any subset?

- (b) What are the sets, if any, that have exactly one subset?
- (c) What are the sets, if any, that have exactly two subsets?
- (d) What are the sets, if any, that have exactly three subsets?

6. Let A be the singleton $\{0\}$, B the pair $\{0,1\}$ and C the triple $\{0,1,2\}$.

(a) Find $B \times C$ and $C \times B$.

(b) Find $B \times A \times C$ and $(B \times A) \times C$.

(c) Find B^3 , $B^2 \times B$ and $B \times B^2$.

7. Consider three sets A, B and C.

(a) When do we have $B \times C = C \times B$?

(b) When do we have $B \times A \times C = (B \times A) \times C$?

8. Whenever possible, express each one of the following sets in the form of an integer interval and in the form of a real interval: \emptyset , $\{1\}$, \mathbb{N} , \mathbb{N}^* , \mathbb{Z} , \mathbb{Z}^- , \mathbb{Z}^* , \mathbb{Z}^+ , \mathbb{R} , \mathbb{R}^- , \mathbb{R}^* , \mathbb{R}^+

PART B (4+2+2+2+2=12 marks)

11. Solve over \mathbb{R} the following equations in x. Explain each step as in Lab 1 Part B; use the properties as listed and numbered in the handout.

(a) $1 - (x+1)^2 = 0$

(b) $1 + \sqrt{x-1} = 0$

(c) $1 + 1/(x+1) = 0$

(d) $1 - |x^2 - 1| = 0$

12. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$

$$x \mapsto 1 - (x+1)^2$$

(a) What is the domain of definition of f?

(b) Explain why the range of f is not \mathbb{R} .

(c) What is (or are) the image(s) of 0?

(d) What is (or are) the preimage(s) of 0?

(e) Assume the codomain of f is \mathbb{R}^* instead of \mathbb{R} . What is then the domain of definition of f?

13. Same questions as above when $f : \mathbb{R} \rightarrow \mathbb{R}$

$$x \mapsto 1 + \sqrt{x-1}$$

14. Same questions as above when $f : \mathbb{R} \rightarrow \mathbb{R}$

$$x \mapsto 1 + 1/(x+1)$$

15. Same questions as above when $f : \mathbb{R} \rightarrow \mathbb{R}$

$$x \mapsto 1 - |x^2 - 1|$$

PART C (2+2+2+2=8 marks)

21. (a) Find the binary expansion of 132.

(b) Find the binary expansion of 13254.

22. (a) Find the decimal expansion of $(1101\ 1111)_2$.

(b) Find the decimal expansion of $(111\ 1011\ 1011\ 1011)_2$.

23. (a) Find the hexadecimal expansion of $(75634120)_8$.

(b) Find the octal expansion of $(AB09B3ACBEF)_{16}$.

24. (a) Find the hexadecimal expansion of $(F07)_{16} + (DAB3E)_{16}$.

(b) Find the octal expansion of $(643)_8 \times (75)_8$.