UNIVERSITY of GUELPH

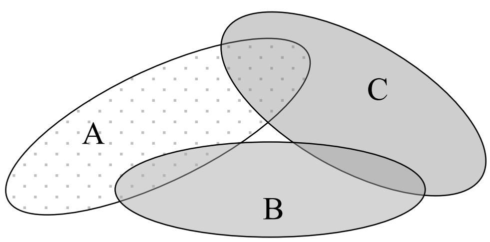
CIS1910 Discrete Structures in Computing (I)
Winter 2019, Solutions to Assignment 1

PART A

1. Possible answers are ***(a)*** S={1,2} ***(b)*** S={0,1} ***(c)*** S={1,{0}} ***(d)*** S={0,{0}}.
For example, the elements of S={1,{0}} are 1 and {0},
i.e., S does not contain 0 but contains {0}.

2. Possible answers are ***(a)*** S={2,3} ***(b)*** S={2,{1}} ***(c)*** S={1,2} ***(d)*** S={1,{1}}.
For example, the elements of S={2,{1}} are 2 and {1}. In particular, {1} belongs to S.
However, since 1 does not belong to S, {1} is not a subset of S.

3.



4. We find ***(a)*** {{}} ***(b)*** {{},{{}}} ***(c)*** {{},{{}},{0},{{},0}}
(d) {{},{{}},{0},{1},{{},0},{{},1},{0,1},{{},0,1}}.
For example, the set {{},0} contains two elements: the empty set {} and the integer 0.
Its subsets are {}, {{}}, {0} and{{},0}.

5. (a) This is not possible: whatever the set S, the empty set {} is a subset of S.
(b) There is only one set like that: the empty set {}.
(c) Any singleton set S has exactly two subsets: {} and S.
(d) A singleton set has 2 subsets (e.g., the subsets of {0} are {} and {0})
and a pair set has 4 (e.g., the subsets of {0,1} are {}, {0}, {1} and {0,1}.
There is no set with exactly 3 subsets.

6. (a) B×C = {(0,0), (0,1), (0,2), (1,0), (1,1), (1,2)}
and C×B = {(0,0), (0,1), (1,0), (1,1), (2,0), (2,1)}
(b) B×A×C = {(0,0,0), (0,0,1), (0,0,2), (1,0,0), (1,0,1), (1,0,2)}
and (B×A)×C = {((0,0),0), ((0,0),1), ((0,0),2), ((1,0),0), ((1,0),1), ((1,0),2)}

(c) $B^3 = B \times B \times B = \{(0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)\}$
and $B^2 \times B = \{((0,0),0), ((0,0),1), ((0,1),0), ((0,1),1), ((1,0),0), ((1,0),1), ((1,1),0), ((1,1),1)\}$
and $B \times B^2 = \{(0,(0,0)), (0,(0,1)), (0,(1,0)), (0,(1,1)), (1,(0,0)), (1,(0,1)), (1,(1,0)), (1,(1,1))\}$

7. (a) $B \times C = C \times B$ iff $B=\{\}$, $C=\{\}$ or $B=C$.
If $B=\{\}$, $C=\{\}$ or $B=C$, we obviously have $B \times C = C \times B$. If not, $B \neq \{\}$, $C \neq \{\}$ and $B \neq C$, and we can show that $B \times C \neq C \times B$: indeed, we can then find an element x that is in B but not C, or vice versa; the two cases are similar, so, without loss of generality, let us assume that $x \in B$ and $x \notin C$; since C is not empty, there is some element $y \in C$; therefore, $(x,y) \in B \times C$, but $(x,y) \notin C \times B$ since $x \notin C$.
(b) $B \times A \times C = (B \times A) \times C$ iff $A=\{\}$, $B=\{\}$ or $C=\{\}$.
If $A=\{\}$, $B=\{\}$ or $C=\{\}$, we obviously have $B \times A \times C = (B \times A) \times C = \{\}$. If not, $A \neq \{\}$, $B \neq \{\}$ and $C \neq \{\}$, and $B \times A \times C \neq (B \times A) \times C$: indeed, the sets $B \times A \times C$ and $(B \times A) \times C$ are not empty then, and while the former is a set of triples, the latter is a set of pairs.

8. $\{\}=1..0=]0,0[$, $\{1\}=1..1=[1,1]$, $\mathbb{N}=0..+\infty$, $\mathbb{N}^*=1..+\infty$,
$\mathbb{Z}=-\infty..+\infty$, $\mathbb{Z}^-=-\infty..-1$, $\mathbb{Z}^*$, $\mathbb{Z}^+=1..+\infty$,
$\mathbb{R}=]-\infty,+\infty[$, $\mathbb{R}^-=]-\infty,0[$, $\mathbb{R}^*$, $\mathbb{R}^+=]0,+\infty[$

PART B

11. We find ***(a)*** $\{0, -2\}$ ***(b)*** $\{\}$ ***(c)*** $\{-2\}$ ***(d)*** $\{-\sqrt{2}, 0, \sqrt{2}\}$.
For example, here is a way to solve the first equation. Let x be a real number.
$1-(x+1)^2=0$ iff $1=(x+1)^2$. This is according to (37) with $a=1-(x+1)^2$, $b=0$, $c=(x+1)^2$.
Moreover, $(x+1)^2=1$ iff $x+1=1$ or $x+1=-1$. This is according to (34) with $a=x+1$, $b=1$.
Finally, according to (37), we have $x+1=1$ iff $x=1-1=0$ and we have $x+1=-1$ iff $x=-1-1=-2$.
In the end, $1-(x+1)^2=0$ iff $x=0$ or $x=-2$.

12. (a) We have to find all the real numbers x such that x belongs to the domain $\mathbb{R}$ and $1-(x+1)^2$ belongs to the codomain $\mathbb{R}$. All real numbers satisfy these conditions. In other words, the domain of definition of f is $\mathbb{R}$.
(b) For example, we can easily show that there is no real number x such that $1-(x+1)^2 = 2$.
In other words, 2 has no preimage under f; it does not belong to the range of f.
(c) Since 0 belongs to the domain of definition of f, it has an image under f: $f(0)=1-(0+1)^2=0$.
(d) According to 11*a*, the preimages of 0 are 0 and -2.
(e) The domain of definition of f is then the set of all real numbers except 0 and -2
(the image of 0, or -2, would be 0, but 0 does not belong to the codomain).

13. (a) $[1,+\infty[$
(b) 0 does not belong to the range of f.
(c) 0 does not have an image under f.
(d) According to 11*b*, the number 0 does not have a preimage under f.
(e) $[1,+\infty[$

14. (a) The set of all real numbers except −1.
(b) 1 does not belong to the range of f.
(c) f(0)=1+1/(0+1)=2
(d) According to 11*c*, the preimage of 0 is −2.
(e) The set of all real numbers except −2 and −1.

15. (a) ℝ
(b) 2 does not belong to the range of f.
(c) f(0)=0
(d) According to 11*d*, the preimages of 0 are −√2, 0 and √2.
(e) The set of all real numbers except −√2, 0 and √2.

PART C

21. We find ***(a)*** $(1000\ 0100)_2$ ***(b)*** $(11\ 0011\ 1100\ 0110)_2$
For example:

←

0	**1**	**2**	**4**	**8**	**16**	**33**	**66**	**132**	**div 2**
	1	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**mod 2**

22. We find ***(a)*** 223 ***(b)*** 31675
For example:

×2+
→

1	**1**	**0**	**1**	**1**	**1**	**1**	**1**
	3	**6**	**13**	**27**	**55**	**111**	**223**

23. We find ***(a)*** $(F73850)_{16}$ ***(b)*** $(253023316545757)_8$
For example, since $(7)_8=(111)_2$, $(5)_8=(101)_2$, $(6)_8=(110)_2$, $(3)_8=(011)_2$, etc., we have $(75634120)_8 = (111\ 101\ 110\ 011\ 100\ 001\ 010\ 000)_2 = (1111\ 0111\ 0011\ 1000\ 0101\ 0000)_2$.
The result follows from the fact that $(1111)_2=(F)_{16}$, $(0111)_2=(7)_{16}$, $(0011)_2=(3)_{16}$, etc.

24. We find ***(a)*** $(DBA45)_{16}$ ***(b)*** $(61727)_8$

D	^{1}**A**	**B**	^{1}**3**	**E**
+		**F**	**0**	**7**
D	**B**	**A**	**4**	**5**

	^{5}	$^{3\,2}$**6**	$^{2\,1}$**4**	**3**
		×	**7**	**5**
^{1}	4	^{1}0	5	7
5	5	6	5	
6	**1**	**7**	**2**	**7**