UNIVERSITY of GUELPH

CIS1910 Discrete Structures in Computing (I)

Winter 2019, Assignment 2

All answers must be justified in a clear, concise and complete manner. If two answers require the same explanations, justify your first answer only, and refer the reader to that justification for the second answer.

PART A (12+4=16 marks)

1. Consider the following binary operations on $\mathbb{R}^2$:

$\oplus : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$
$((a,b),(c,d)) \mapsto (ad+bc,bd)$

$\otimes : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$
$((a,b),(c,d)) \mapsto (ac,bd)$

(a) Show that $\otimes$ is commutative.
(b) Show that $\oplus$ is associative.
(c) Show that $\otimes$ is not distributive over $\oplus$.
(d) Show that there is a neutral element for $\oplus$.

2. Let $\star$ be a binary operation on a set S. Assume the domain of definition of $\star$ is S^2, assume $\star$ is associative and n is the neutral element for $\star$. Now, let s and t be two elements of S. We say that t is a ***left inverse*** of s under $\star$ iff $t \star s = n$. We say that t is a ***right inverse*** of s iff $s \star t = n$. Show that if an element of S has both a left inverse and a right inverse under $\star$ then the left inverse and the right inverse are equal.

PART B (3.5+8.5=12 marks)

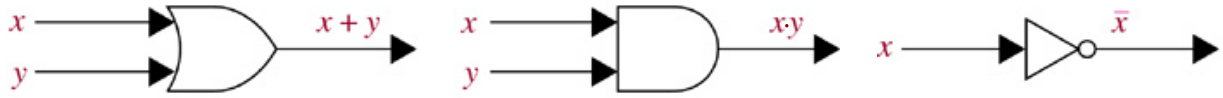
Consider the Boolean algebra $(\{0,1\},+,\cdot,^-)$ as in slide 2.11. The Boolean operations +, · and $^-$ can then be defined by the tables below. Here, the symbol + reads "or" instead of "plus", the symbol · reads "and" instead of "dot", and the symbol $^-$ reads "not" instead of "bar".

x	y	x+y
0	0	0
0	1	1
1	0	1
1	1	1

x	y	x·y
0	0	0
0	1	0
1	0	0
1	1	1

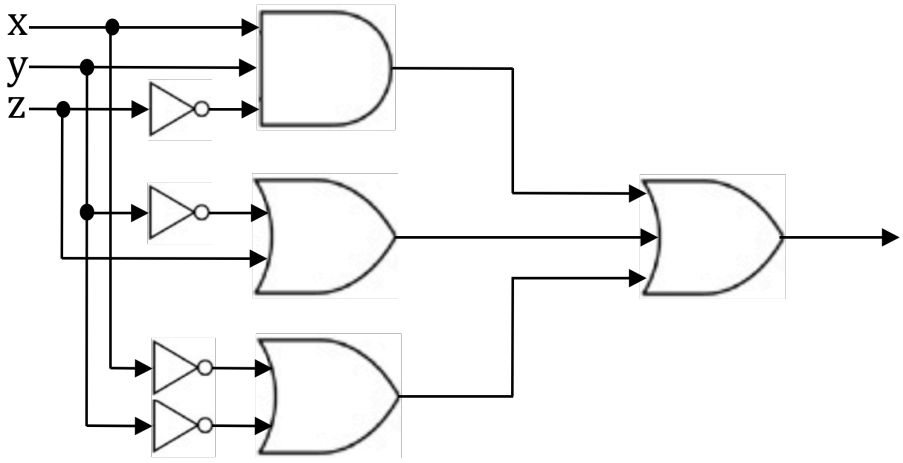
x	$\overline{x}$
0	1
1	0

This Boolean algebra is at the basis of circuit design. A computer is made up of a number of circuits. The basic elements of circuits are gates. Typically, there are one or more inputs to a gate, and only one output. Gate inputs are driven by voltages having two nominal values (e.g., 0V and 5V); these values are represented by the symbols 0 and 1 respectively. The output of a gate also provides two nominal values of voltage only. Common gates are:



OR gate **AND** gate **INVERTER**

11. What is the output to the circuit below?



12. Construct circuits to produce the following outputs:

(a) $x\cdot\overline{y}$
(b) $x+\overline{x\cdot y}$
(c) $(x+y+\overline{z})\cdot(\overline{x}+\overline{y}+z)$
(d) $\overline{(x\cdot\overline{y})+(y\cdot\overline{z})}$

PART C (3+4+3+5+4+4=23 marks)

When answering the questions below, be aware that there are different ways to express $p \rightarrow q$. *For example:*

"if p, then q"	"q if p"	"p is sufficient for q"
"if p, q"	"q when p"	"a sufficient condition for q is p"
"p implies q"	"q unless not p"	"q is necessary for p"
"p only if q"	"q follows from p"	"a necessary condition for p is q"

There are also different ways to express $p \leftrightarrow q$.

21. Which of these are propositions? What are the truth values of those that are propositions?

(a) $\sqrt{n}=2$.
(b) Consider an integer n: $\sqrt{n}=2$ and $n=4$.
(c) Consider an integer n: if $\sqrt{n}=2$ then $n=4$.
(d) How are you doing?
(e) Most dogs fly.
(f) Be brave.

22. Let p be the proposition "I read the newspapers" and let q be the proposition "I get depressed". Translate each of the following propositional expressions into English: ***(a)*** $\neg p$, ***(b)*** $p \vee q$, ***(c)*** $p \rightarrow q$, ***(d)*** $p \wedge q$, ***(e)*** $p \leftrightarrow q$, ***(f)*** $(\neg p) \rightarrow (\neg q)$, ***(g)*** $(\neg p) \wedge (\neg q)$, ***(h)*** $(\neg p) \vee (p \wedge q)$.

23. Let p be the proposition "He was very active",
let q be the proposition "He had a healthy diet"
and let r be "He lived for over 100 years".
Translate into propositional expressions:

(a) Being very active and having a healthy diet are the reasons why he lived for over 100 years.
(b) If he lived for over 100 years it is because and only because he had a healthy diet.
(c) He was very active, he had a healthy diet, and he lived for over 100 years.
(d) He lived for over 100 years, but he did not have a healthy diet.
(e) To live for over 100 years, he had to be very active.
(f) He was very active, but he did not have a healthy diet;
nevertheless, he lived for over 100 years.

24. Translate each of the following statements into a propositional expression involving exactly two propositions, p and q. In each case, specify what p and q are.

(a) It is necessary to wash the boss' car to get promoted.
(b) Winds from the south imply a spring thaw.
(c) If you read the newspaper every day, you will be informed, and conversely.
(d) A sufficient condition for the warranty to be good
is that you bought the computer less than a year ago.
(e) Willy gets caught whenever he cheats.
(f) You can access the website only if you pay a subscription fee.
(g) You can see the wizard only if the wizard is not in,
and the wizard is not in only if you can see him.
(h) Getting elected follows from knowing the right people.
(i) Carol gets seasick whenever she is on a boat.
(j) It rains if it is a weekend day, and it is a weekend day if it rains.

25. True or false?

(a) If flies are insects then elephants are mammals.
(b) Flies are insects iff elephants are mammals.
(c) If 1+1=3 then elephants are mammals.
(d) If elephants are mammals then 1+1=3.
(e) 1+1=3 iff elephants are mammals.
(f) Elephants are mammals iff 1+1=3.
(g) If flies are mammals then 1+1=3.
(h) Flies are mammals iff 1+1=3.

26. Construct truth tables for the following propositional expressions:

(a) $((p\rightarrow q)\wedge(q\rightarrow r))\rightarrow(p\rightarrow r)$
(b) $((p\rightarrow q)\wedge(q\rightarrow r))\rightarrow(p\leftrightarrow r)$
(c) $((p\leftrightarrow q)\wedge(q\leftrightarrow r))\rightarrow(p\leftrightarrow r)$
(d) $((p\leftrightarrow q)\wedge(q\rightarrow r))\rightarrow(p\rightarrow r)$