CIS1910 Discrete Structures in Computing (I)
Winter 2019, Solutions to Assignment 2

PART A

1. Let (a,b), (c,d) and (e,f) be three elements of $\mathbb{R}^2$.

(a) We have: $(a,b)\otimes(c,d) = (ac,bd)$ (according to the definition of $\otimes$)
Moreover: $(c,d)\otimes(a,b) = (ca,db)$ (according to the definition of $\otimes$)
$= (ac,bd)$ (since $\times$ is commutative)
In the end, $(a,b)\otimes(c,d) = (c,d)\otimes(a,b)$. Q.E.D.

(b) $((a,b)\oplus(c,d))\oplus(e,f) = (ad+bc,bd)\oplus(e,f) = ((ad+bc)f+bde,bdf)$
$= (adf+bcf+bde,bdf)$
$(a,b)\oplus((c,d)\oplus(e,f)) = (a,b)\oplus(cf+de,df) = (adf+b(cf+de),bdf)$
$= (adf+bcf+bde,bdf)$
In the end, $((a,b)\oplus(c,d))\oplus(e,f) = (a,b)\oplus((c,d)\oplus(e,f))$. Q.E.D.

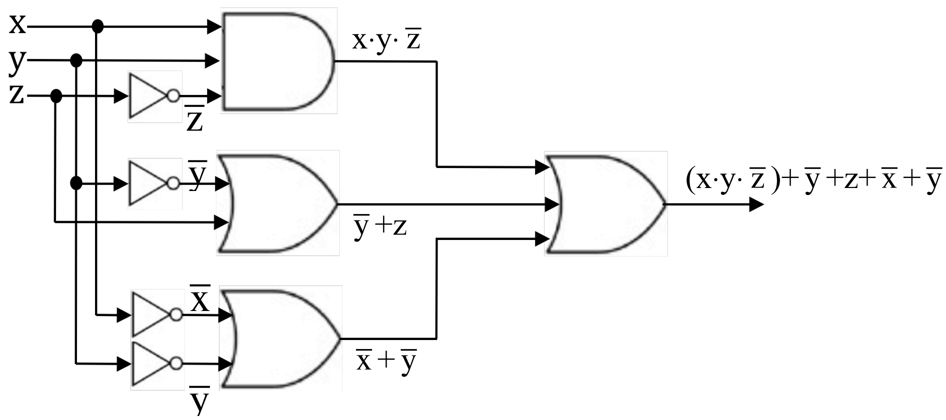
(c) For example, we have:
$(0,2)\otimes((0,1)\oplus(0,1)) = (0,2)\otimes(0,1) = (0,2)$
and $((0,2)\otimes(0,1))\oplus((0,2)\otimes(0,1)) = (0,2)\oplus(0,2) = (0,4)$
Since $(0,2)\otimes((0,1)\oplus(0,1)) \neq ((0,2)\otimes(0,1))\oplus((0,2)\otimes(0,1))$,
$\otimes$ is not distributive over $\oplus$. Q.E.D.

(d) It is easy to check that (0,1) is a neutral element for $\oplus$:
$(a,b)\oplus(0,1) = (0,1)\oplus(a,b) = (a,b)$ Q.E.D.

2. Let s, t and t’ be elements of S. Assume t is a left inverse of s and t’ is a right inverse.
We have: $(t\star s)\star t' = n\star t' = t'$.
Moreover, since $\star$ is associative: $(t\star s)\star t' = t\star(s\star t') = t\star n = t$
In the end, t=t’. Q.E.D.

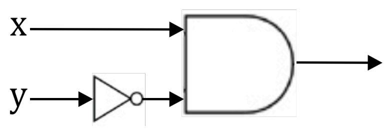
PART B

11.

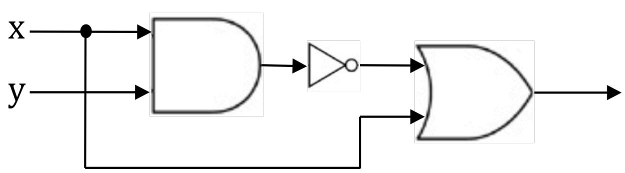


12.

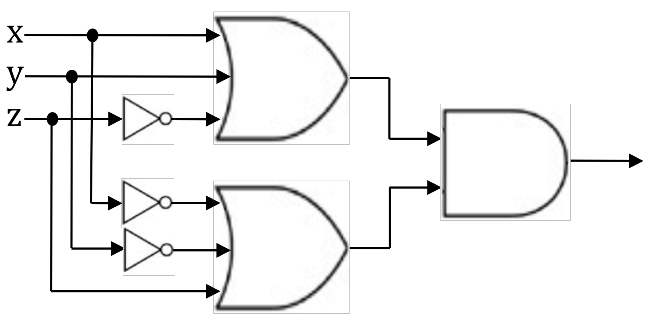
(a)



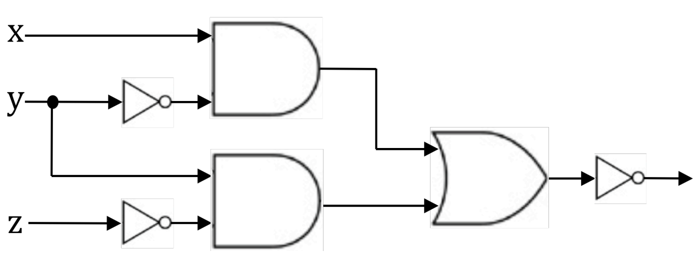
(b)



(c)



(d)



PART C

21. (a)(d)(f) are not propositions, but ***(b)(c)(e)*** are; the truth value of ***(b)*** is **F**, the truth value of ***(c)*** is **T**, and the truth value of ***(e)*** is **F**.

22. (a) I don't read the newspapers.
(b) I read the newspapers, or I get depressed.
(c) If I read the newspapers then I get depressed.
(d) I read the newspapers and I get depressed.
(e) I read the newspapers if and only if I get depressed.
(f) If I don't read the newspapers then I don't get depressed.
(g) I don't read the newspapers and I don't get depressed.
(h) I don't read the newspapers, or I read them and I get depressed.

23. (a) $(p\wedge q)\rightarrow r$ ***(b)*** $r\leftrightarrow q$ ***(c)*** $p\wedge q\wedge r$ ***(d)*** $r\wedge\neg q$ ***(e)*** $r\rightarrow p$ ***(f)*** $p\wedge(\neg q)\wedge r$

24. (a) $p\rightarrow q$ with p="you get promoted" and q="you wash the boss' car"
(b) $p\rightarrow q$ with p="the winds are from the south" and q="there will be a spring thaw"
(c) $p\leftrightarrow q$ with p="you will be informed" and q="you read the newspaper every day"
(d) $p\rightarrow q$ with p="you bought the computer less than a year ago" and q="the warranty is good"
(e) $p\rightarrow q$ with p="Willy cheats" and q="Willy gets caught"
(f) $p\rightarrow q$ with p="you access the website" and q="you must pay a subscription fee"
(g) $p\leftrightarrow q$ with p="you can see the wizard" and q="the wizard is not in"
(h) $p\rightarrow q$ with p="you know the right people" and q="you will be elected"
(i) $p\rightarrow q$ with p="Carol is on the boat" and q="Carol gets seasick"
(j) $p\leftrightarrow q$ with p="it rains" and q="it is a weekend day"

25. (d)(e)(f) are false, while ***(a)(b)(c)(g)(h)*** are true.

26. (a) The propositional expression $((p\rightarrow q)\wedge(q\rightarrow r))\rightarrow(p\rightarrow r)$ is always true, as shown below. We say that it is a ***tautology***.

p	q	r	$p\rightarrow q$	$q\rightarrow r$	$p\rightarrow r$	$(p\rightarrow q)\wedge(q\rightarrow r)$	$((p\rightarrow q)\wedge(q\rightarrow r))\rightarrow(p\rightarrow r)$
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	T	F	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T

(c) and ***(d)*** are tautologies as well, but ***(b)*** is not:

p	q	r	p→q	q→r	p↔r	(p→q)∧(q→r)	((p→q)∧(q→r))→(p↔r)
F	F	F	T	T	T	T	T
F	F	T	T	T	F	T	F
F	T	F	T	F	T	F	T
F	T	T	T	T	F	T	F
T	F	F	F	T	F	F	T
T	F	T	F	T	T	F	T
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T