CIS1910 Discrete Structures in Computing (I)
Winter 2019, Assignment 3

All answers must be justified in a clear, concise and complete manner.

A. Rules of Inference (2+1.5+3+4.5 marks)

1) Prove that the premises “I am either dreaming or hallucinating”, “I am not dreaming” and “If I am hallucinating, I see elephants running down the road” imply the conclusion “I see elephants running down the road”.

2) Prove that the premises “Allen is a bad boy or Hillary is a good girl” and “Allen is a good boy or David is happy” imply the conclusion “Hillary is a good girl or David is happy”.

3) Prove that the premises $(p\wedge t)\rightarrow(r\vee s)$, $q\rightarrow(u\wedge t)$, $u\rightarrow p$, $\neg s$ and q imply the conclusion r.

4) Prove that the premises “There is someone in this class who has been to France” and “Everyone who goes to France visits the Louvre” imply the conclusion “Someone in this class has visited the Louvre”.

B. Direct Proofs (2+2 marks)

5) Show that the product of two odd integers is odd.

6) Consider two integers *a* and *b*, with $a\neq0$. We say that *a* ***divides*** *b*, and we write $a|b$, iff there exists an integer *m* such that $b=am$. Now, consider three integers *a*, *b* and *c*, with $a\neq0$ and $b\neq0$; show that if $a|b$ and $b|c$ then $a|c$.

C. Proofs by Contraposition (2+2 marks)

7) Consider two real numbers x and y.
Show that if the product xy is an irrational number then x or y is an irrational number.

8) Prove that for any integer n, if n^5+7 is even then n is odd.

D. Proofs by Contradiction (3+3 marks)

9) Let a, b and c be three integers, with $a \neq 0$.
Show that if a does not divide bc then a does not divide b.

10) Show that for any nonnegative real numbers a and b we have: $(a+b)/2 \geq \sqrt{(ab)}$

E. Proofs by Induction (4+4 marks)

11) The ***factorial*** of a positive integer n, denoted by n!, is the product of all positive integers less than or equal to n. For example, $1!=1$, $2!=2\times1=2$ and $3!=3\times2\times1=6$. Find a formula for $1\times1!+2\times2!+\ldots+n\times n!$ by examining the values of this expression for small values of n; then, prove the formula.

12) Show that for any positive odd integer n the number n^2-1 is divisible by 8.

F. Proofs by Strong Induction (5+5 marks)

13) Assume that a chocolate bar consists of n squares arranged in a rectangular pattern. The entire bar, or any smaller rectangular piece of the bar, can be broken along a vertical or a horizontal line separating the squares. Assuming that only one piece can be broken at a time, determine how many breaks you must successively make to break the bar into n separate squares.

14) For any positive integer n, let T_n be the number 1 if $n<4$ and the number $T_{n-1}+T_{n-2}+T_{n-3}$ if $n\geq4$.
We have $T_1=1$, $T_2=1$, $T_3=1$, $T_4=T_3+T_2+T_1=1+1+1=3$, $T_5=T_4+T_3+T_2=3+1+1=5$, etc. Prove that:
$\forall n\in\mathbb{Z}^+, T_n<2^n$.

G. Other (2+2+4 marks)

15) Prove that for all real numbers x and y we have: $|x+y|\leq|x|+|y|$

16) Show that for any real numbers x and y such that $x<y$
there exists a real number z such that $x<z<y$.

17) Write a C program to prove that there is no positive integer less than 10
whose cube is the sum of the cubes of two positive integers. Explain.