CIS1910 Discrete Structures in Computing (I)
Winter 2019, Lab 10 Notes

Here are recommended practice exercises on binary relations and functions.
Many of these exercises have been covered in Lab 10.

QUESTION A.

Consider the following definition and notation:

Let $\Re$ be the triple (U,V,G), where U and V are two sets and G is a subset of U×V.
We then say that $\Re$ is a ***binary relation over*** U ***and*** V.

Let (u,v) be an element of G.
We then say that u is related to v by $\Re$, and we write **u$\Re$v**.

1.

Come up with a few examples of binary relations and represent each one by a diagram as you would with a function.

QUESTION B.

Consider the following definition:

The ***inverse*** of the binary relation $\Re$=(U,V,G) is the relation $\Re^{-1}$=(V,U,G^{-1}) where G^{-1}={(v,u)∈ V×U | (u,v)∈ G}.

2.

Represent by a diagram the inverse of each one of the binary relations found in **Question A**.

QUESTION C.

Consider the following definitions:

Note that a ***function*** from U to V is a binary relation over U and V such that (see slide 1.16):

$$\forall u \in U,\ \forall (v_1,v_2) \in V^2,\ [(u,v_1) \in G \wedge (u,v_2) \in G] \rightarrow v_1=v_2$$

We say that a function f from U to V is ***total*** iff the domain of definition of f is U.
f is ***surjective*** iff the range of f is V.
f is ***injective*** iff the inverse f^{-1} of the (function, and hence) relation f is a function.
f is ***bijective*** iff f is total, surjective and injective.

3.

What are all the binary relations over the pair sets $\{a,b\}$ and $\{0,1\}$? Which relations are functions, and which functions are total, injective, surjective, bijective? Represent each relation by a diagram and write the words "function", "total", "surjective", "injective", "bijective" under the diagram, as appropriate.

4.

Consider the function

$$f : \mathbb{R} \rightarrow \mathbb{R}, \quad x \mapsto 3x-2$$

(a) What is the domain of definition of f?
(b) Let y be an element of the codomain of f. Solve the equation $f(x)=y$ in x.
Note that you might have to consider different cases, depending on y.
(c) What is the range of f?
(d) Is f total, surjective, injective, bijective?
If f is injective, what is its inverse f^{-1}?

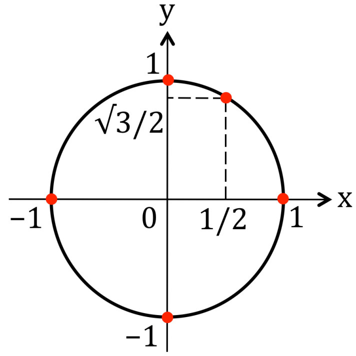
5.

Same questions as above when f is the function

$$\mathbb{R} \rightarrow \mathbb{R}, \quad x \mapsto 3+(x-2)^2$$

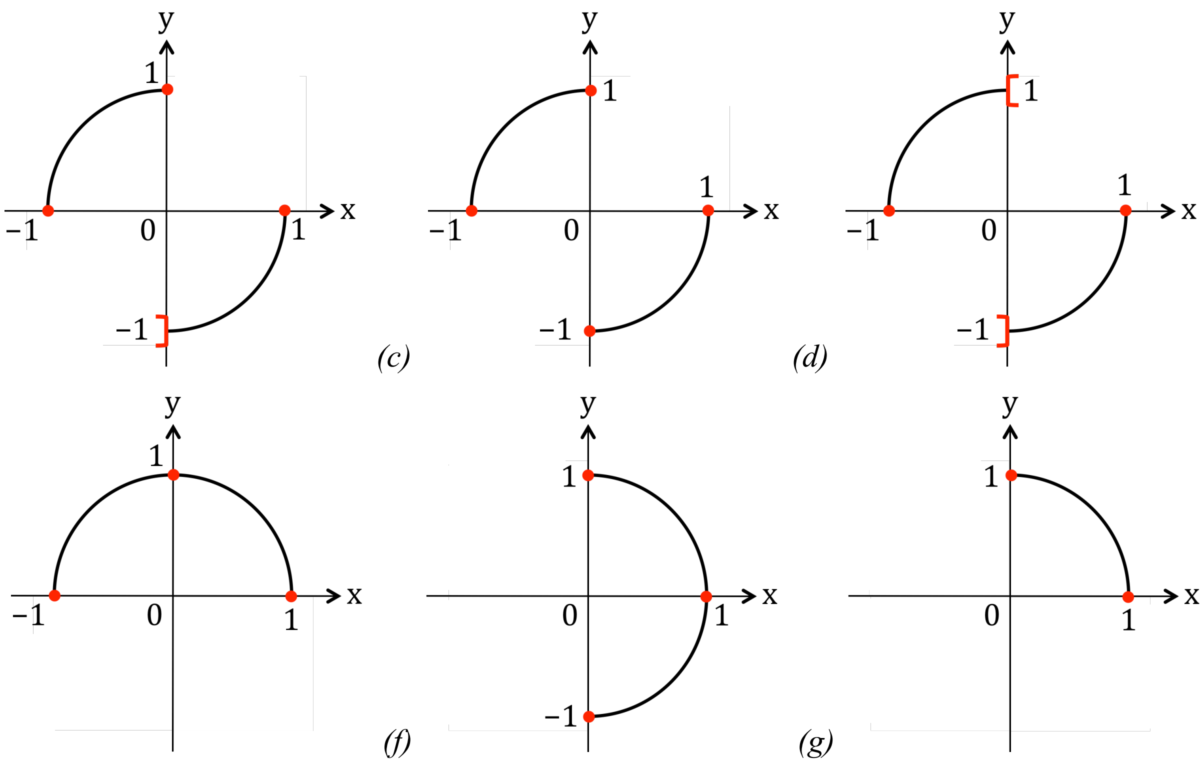
QUESTION D.

Consider the binary relation $\Re$ over $[-1,1]$ and $[-1,1]$ defined by: $x \Re y \leftrightarrow x^2+y^2=1$.
This relation can be plotted as shown below. Note that, according to this plot,
$-1 \Re 0$, $1 \Re 0$, $0 \Re -1$, $0 \Re 1$, and $1/2 \ \Re \ \sqrt{3}/2$.



(a)

The following plots represent various binary relations over real intervals. Note that in *(b)*, the square bracket turns its "back" on the curve and indicates that 0 is not related to −1; in *(d)*, the brackets indicate that 0 is not related to −1 and 0 is not related to 1.



(b) (c) (d) (e) (f) (g)

6.

Is the binary relation over [−1,1] and [−1,1] represented by the plot *(a)* a function?
If the answer is yes, is this function total, injective, surjective, bijective?
Same questions with the plots *(b)*, *(c)*, *(d)*, *(e)*, *(f)* and *(g)*.

7.

Is the binary relation over $[-1,1]$ and $[0,1]$ represented by the plot *(e)* a function?
If the answer is yes, is this function total, injective, surjective, bijective?

8.

Is the binary relation over $[0,1]$ and $[-1,1]$ represented by the plot *(f)* a function?
If the answer is yes, is this function total, injective, surjective, bijective?

9.

Is the binary relation over $[0,1]$ and $[0,1]$ represented by the plot *(g)* a function?
If the answer is yes, is this function total, injective, surjective, bijective?

QUESTION E.

Consider the following definitions:

Let U be a set and let $\Re$ be a binary relation over U and U.
We then say that $\Re$ is a ***binary relation on*** U.

$\Re$ is ***reflexive*** iff:	$\forall u \in U, u \Re u$
$\Re$ is ***symmetric*** iff:	$\forall (u,v) \in U^2, (u \Re v \rightarrow v \Re u)$
$\Re$ is ***antisymmetric*** iff:	$\forall (u,v) \in U^2, [(u \Re v \wedge v \Re u) \rightarrow u=v]$
$\Re$ is ***transitive*** iff:	$\forall (u,v,w) \in U^3, [(u \Re v \wedge v \Re w) \rightarrow u \Re w]$

10.

Consider the binary relation $\Re$ on the set $\mathbb{R}$ of all real numbers defined by: $x \Re y \leftrightarrow x+y=0$.

(a) Give some examples of elements that are related by $\Re$.
(b) Is $\Re$ reflexive, symmetric, antisymmetric, transitive?

11.

Same as above when $\Re$ is the binary relation on $\mathbb{R}$ defined by:
$x \Re y \leftrightarrow (x=y \vee x=-y)$.

12.

Same as above when $\Re$ is the binary relation on $\mathbb{N}^2$ (where $\mathbb{N}$ is the set of natural numbers) defined by: $(u,v)\Re(u',v') \leftrightarrow u+v'=u'+v$.

QUESTION F.

Consider the following definition:

A ***partition*** P of a set S is a set of subsets of S such that:

any element of P is a nonempty set,
any two distinct elements of P are disjoint,
and any element of S belongs to some element of P.

13.

Find all possible partitions of {1,2,3}.
For each partition, draw a diagram that shows how the set {1,2,3} is partitioned.