UNIVERSITY of GUELPH

CIS1910 Discrete Structures in Computing (I)
Winter 2019, Lab 11 Notes

Here are other recommended practice exercises on binary relations and functions. Many of these exercises have been covered in Lab 11. Note that the examples and exercises listed in blue come from the following textbook: "Discrete Mathematics and Its Applications," by Rosen, Mc Graw Hill, 7th Edition. *Be aware that there are some discrepancies between the lecture notes and this textbook: the definitions and notations are not always exactly the same.*

PART A — ORDER RELATIONS AND HASSE DIAGRAMS

Types of questions you should be able to answer:

☐ Given an order relation on a finite set, draw the corresponding digraph representation.

☐ Given the digraph representation of an order relation, draw the corresponding Hasse diagram.

☐ Given a Hasse diagram, draw the corresponding digraph representation.

☐ Given the Hasse diagram of an ordered set, find the maxima, the minima, the greatest element and the least element (if any) of that ordered set.

☐ Given the Hasse diagram of an ordered set $(U, \lhd)$ and some subset V of U, find the upper bounds, the lower bounds, the supremum and infimum of V (if any).

Examples of digraph representations of order relations:

See textbook, Figs. 2a and 3a pp. 623-4; Exercise 11 p. 630.

Examples of ordered sets and corresponding Hasse diagrams:

$(\{1,2,3,4\},\leq)$; see textbook, Fig. 2c p. 623.
$(\{1,2,3,4,6,8,12\},|)$ where | denotes the relation "divides"; see Fig. 3c p. 624.
$(2^{\{a,b,c\}},\subseteq)$; see Fig. 4 p. 624.
$(\{2,4,5,10,12,20,25\},|)$; see Fig. 5 p. 624.

Other examples of Hasse diagrams:

See textbook, Figs. 6-8 pp. 625-6; Exercises 25-27, 32 and 43 pp. 631-2.

Examples of digraph representations of binary relations that are not order relations:

See textbook, Exercises 9-10 p. 630.

PART B — COMPOSITION AND SUM OF FUNCTIONS

1.

Consider a function f from U to V and a function g from V to W. Since f and g are also binary relations, we can define the composition g∘f of f and g, and it is easy to show that g∘f is a function. Let D_f be the domain of definition of f and let D_g be the domain of definition of g. What is the domain of definition of g∘f? (Use set builder notation.)

2.

Consider the following functions:

$$f : \mathbb{R} \to \mathbb{R} \qquad\qquad g : \mathbb{R} \to \mathbb{R}$$
$$x \mapsto 2x-1 \qquad\qquad x \mapsto x^2$$

Define g∘f and f∘g.

3.

Things may become a bit tricky when the functions are not from ℝ to ℝ, or are not total.
Consider the following functions:

$$f : \mathbb{R} \to \mathbb{R}^+ \qquad\qquad g : \mathbb{R}^+ \to \mathbb{R}$$
$$x \mapsto 2x-1 \qquad\qquad x \mapsto x^2$$

Define g∘f and f∘g.

4.

Consider the functions f and g from ℝ to ℝ defined by $f(x)=\sqrt{x}$ and $g(x)=x^2$.
Define g∘f and f∘g.

5.

Consider two functions f and g from U to V, where V is a subset of $\mathbb{R}$.
We can then define the functions

$$\begin{aligned} f+g &: U \to V \\ x &\mapsto f(x)+g(x) \end{aligned} \qquad \begin{aligned} fg &: U \to V \\ x &\mapsto f(x)g(x) \end{aligned} \qquad \begin{aligned} f-g &: U \to V \\ x &\mapsto f(x)-g(x) \end{aligned} \qquad \text{etc.}$$

Let D_f be the domain of definition of f and let D_g be the domain of definition of g.
What is the domain of definition of f+g? (Use set builder notation.)

6.

Consider the following functions:

$$\begin{aligned} f &: \mathbb{R} \to \mathbb{R} \\ x &\mapsto x^2-x \end{aligned} \qquad \begin{aligned} g &: \mathbb{R} \to \mathbb{R} \\ x &\mapsto 1+x \end{aligned}$$

Define f+g.

7.

Again, things may become a bit tricky when the functions are not from $\mathbb{R}$ to $\mathbb{R}$, or are not total.
Consider the following functions:

$$\begin{aligned} f &: \mathbb{R} \to \mathbb{R} \\ x &\mapsto x^2-\sqrt{x} \end{aligned} \qquad \begin{aligned} g &: \mathbb{R} \to \mathbb{R} \\ x &\mapsto 1+\sqrt{x} \end{aligned}$$

Define f+g.

8.

Consider the following functions:

$$\begin{aligned} f &: \mathbb{R} \to [0,1] \\ x &\mapsto 1-2x \end{aligned} \qquad \begin{aligned} g &: \mathbb{R} \to [0,1] \\ x &\mapsto 3x-1 \end{aligned}$$

Define f+g.