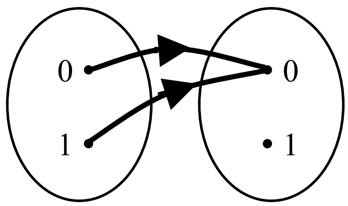
CIS1910 Discrete Structures in Computing (I)
Winter 2019, Lab 3 Notes

Here are recommended practice exercises. Many have been covered in Lab 3.

I.

1. The unary operation ({0,1},{0,1},{(0,0),(1,0)}) can be represented by a diagram, but it can also be represented by a table:



element	image
0	0
1	0

Represent each possible unary operation on {0,1} by a table.

2. What are all the binary operations on {0,1}?

3. Let n be a positive integer. How many n-ary operations can we define on {0,1}?

II.

Consider a binary operation ★ on a set S.

1. Assume x, y and z are three elements of S such that x★y=x★z. Can we then write y=z?

2. Let u and v be two elements of S. Can we write u★v=v★u?

3. Let u, v and w be three elements of S. Can we write u★(v★w)=(u★v)★w?

4. Assume n is an element of S such that:

for any u in S we have u★n=n★u=u.

As seen in class, we then say that n is a ***neutral element*** for ★.
Show that no other element of S can be a neutral element for ★.

5. Assume *a* is an element of S such that:

for any u in S we have u★*a*=*a*★u=*a*.

As seen in class, we then say that *a* is an ***absorbing element*** for ★.
Show that no other element of S can be an absorbing element for ★.

III.

Consider a set B, a unary operation $\overline{}$ on B, and two binary operations + and · on B.
Assume 0 is the neutral element for + and 1 is the neutral element for ·. For any x of B we have:

$$x+0=0+x=x \qquad (1a)$$
$$x\cdot 1=1\cdot x=x \qquad (1b)$$

Assume, moreover, that for any x, y and z of B we have:

$$x+y=y+x \qquad (2a)$$
$$x\cdot y=y\cdot x \qquad (2b)$$

$$(x+y)+z=x+(y+z) \qquad (3a)$$
$$(x\cdot y)\cdot z=x\cdot(y\cdot z) \qquad (3b)$$

$$x+(y\cdot z)=(x+y)\cdot(x+z) \qquad (4a)$$
$$(x\cdot y)+z=(x+z)\cdot(y+z) \qquad (4b)$$

$$x\cdot(y+z)=(x\cdot y)+(x\cdot z) \qquad (4c)$$
$$(x+y)\cdot z=(x\cdot z)+(y\cdot z) \qquad (4d)$$

$$x+\overline{x}=\overline{x}+x=1 \qquad (5a)$$
$$x\cdot\overline{x}=\overline{x}\cdot x=0 \qquad (5b)$$

0. ***(a)*** (1a) means that 0 is the neutral element for + and (1b) that 1 is the neutral element for ·. What about (2a) to (4d)? ***(b)*** If · is given a higher precedence than +, some brackets in these equalities become unnecessary. Which ones? ***(c)*** If · and + are given the same precedence and left-to-right associativity is assumed, some brackets in these equalities become unnecessary. Which ones?

1. Show that $0+0=1\cdot 0=0\cdot 1=0$ and $1+1=1\cdot 1=1$.

2. Show that $1=\overline{0}$.

3. Show that for any two elements x and y of B, if $x+y=1$ and $x\cdot y=0$ then $y=\overline{x}$.

4. Show that for any x of B we have $x=\overline{\overline{x}}$.

5. Show that for any x of B we have $x+x=x$.

6. Show that for any x of B we have $1+x=x+1=1$.

7. Show that for any x of B we have $0\cdot x=x\cdot 0=0$.

8. Show that for any two elements x and y of B we have $\overline{x}\cdot\overline{y}=\overline{x+y}$.