CIS1910 Discrete Structures in Computing (I)
Winter 2019, Lab 5 Notes

Here are recommended practice exercises. Many have been covered in Lab 5.
Note that the examples and exercises listed in blue come from the following textbook:
"Discrete Mathematics and Its Applications," by Rosen, Mc Graw Hill, 7th Edition

A. BOOLEAN ALGEBRA

Consider a Boolean algebra $(B, +, \cdot, \overline{})$.
Assume $\cdot$ is given a higher precedence than $+$.

1. Prove that for any elements x and y of B we have $x+x\cdot y=x$ and $x\cdot(x+y)=x$.

2. Remember that $\downarrow$ (read "***nor***") is the Boolean operation defined by $x\downarrow y = \overline{x+y}$,
for any elements x and y of B. Show that for any x and y of B we have:
$\overline{x}=x\downarrow x$ and $x\cdot y=(x\downarrow x)\downarrow(y\downarrow y)$ and $x+y=(x\downarrow y)\downarrow(x\downarrow y)$.

3. Show that for any elements x and y of B we have: $\overline{x}\cdot\overline{y} + \overline{x}\cdot y + x\cdot\overline{y} = \overline{x}+\overline{y}$

4. Assume B={0,1}. Consider the Boolean operation | (read "***nand***") defined by the table below.

x	y	x | y
0	0	1
0	1	1
1	0	1
1	1	0

(a) Is | idempotent? Is it commutative? Is it associative?
Is there a neutral element for |? Is there an absorbing element for |?

(b) What is the sum-of-products expansion of |?
What is its product-of-sums expansion?

5. Assume B={0,1}. Consider the Boolean function *F* defined by the table below.
What is the sum-of-products expansion of *F* ? What is its product-of-sums expansion?

x	*y*	*z*	*F(x,y,z)*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

B. PROPOSITIONAL LOGIC

1. Example 9 from Section 1.1 of the textbook

2. Examples 3, 6, 7 and 8 from Section 1.3 of the textbook

3. ***(a)*** System specifications should be ***consistent***, that is, they should not contain conflicting requirements. In other words, there should be at least one assignment of truth values that makes all specifications true. When specifications are not consistent, there would be no way to develop a system that satisfies all specifications.

Examples 4 and 5 from Section 1.2 of the textbook

(b) Propositional operations such as $\neg$, $\wedge$, $\vee$ are used extensively in searches of large collections of information such as indexes of Web pages. These searches employ techniques from propositional logic and are called ***Boolean searches***.

Example 6 from Section 1.2 of the textbook

(c) Puzzles that can be solved using logical reasoning are known as ***logic puzzles***. Solving logic puzzles is an excellent way to practice working with the rules of logic. Also, computer programs designed to carry out logical reasoning often use well-known logic puzzles to illustrate their capabilities.

Example 8 from Section 1.2 of the textbook