UNIVERSITY of GUELPH

CIS1910 Discrete Structures in Computing (I)

Winter 2019, Lab 6 Notes

Here are recommended practice exercises. Many have been covered in Lab 6.

PART A.

Propositional and predicate expressions are examples of mathematical expressions. While English sentences are made of words, mathematical expressions are made of mathematical symbols. Translating English sentences into mathematical expressions and vice versa is a crucial task in mathematics, logic programming, artificial intelligence, software engineering, and many other disciplines.

1. Translate into predicate expressions.

(a) "There is an honest politician in Canada."
(b) "Every student in this class has studied calculus."
(c) "Every real number is negative, positive, or equal to 0."

2. Translate into English.

(a) $\forall a, \mathrm{C}(a)$
where C is the predicate defined by

$$\begin{aligned} \mathrm{C} : \mathrm{A} &\to \mathscr{P} \\ a &\mapsto \text{"}a\text{ eats cheeseburgers"} \end{aligned}$$

and A is the set of all Americans.

(b) $\forall \mathrm{p}, (\mathrm{S}(\mathrm{p}) \to (\mathrm{F}(\mathrm{p}) \lor \mathrm{G}(\mathrm{p})))$
where S, F and G are the predicates defined by

$\mathrm{S} : \mathrm{P} \to \mathscr{P}$	$\mathrm{F} : \mathrm{P} \to \mathscr{P}$	$\mathrm{G} : \mathrm{P} \to \mathscr{P}$
$\mathrm{p} \mapsto$ "p is a student in this class"	$\mathrm{p} \mapsto$ "p has visited France"	$\mathrm{p} \mapsto$ "p has visited Greece"

and P is the set of all people.

PART B.

Here, the translations involve nested quantifiers.

1. Translate into mathematical expressions.

(a) "The sum of two positive integers is always positive."
(b) "If a person is female and is a parent, then this person is someone's mother."

2. Translate into English.

(a) $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}, [\ (x>0 \wedge y<0) \rightarrow xy<0\ ]$

(b) $\forall s, [\ C(s) \vee \exists t, (C(t) \wedge F(s,t))\ ]$
where C and F are the predicates defined by

$$C : S \rightarrow \mathscr{P} \qquad s \mapsto \text{"s has a computer"}$$

$$F : S^2 \rightarrow \mathscr{P} \qquad (s,t) \mapsto \text{"s and t are friends"}$$

and S is the set of all students.

PART C.

1. In Lab 1 Part B, we discussed many properties that are useful for solving equations.
For example, for any real numbers a, b and c, we have:

(a) If $a=b$ then $a^2=b^2$.
(b) If $a=b$ and $a\neq 0$ then $1/a=1/b$.
(c) If $a^2=b^2$ then $a=b$ or $a=-b$.
(d) $1/a=1/b$ iff $a=b$ and $a\neq 0$.
(e) $a^2=b^2$ iff $a=b$ or $a=-b$.
(f) $|a|=b$ iff ($a=b$ or $a=-b$) and $b\geq 0$.
(g) $a/c=b/c$ iff $a=b$ and $c\neq 0$.

Express the properties above mathematically, i.e., do not use any English words;
use mathematical symbols instead.

2. Let S be a nonempty set and let $\perp$ (read "thing") be a binary operation on S
(assume its domain of definition is S^2). Express the statements below mathematically,
and specify the binary and unary predicates implicitly associated with the resulting expressions.

(a) $\perp$ is commutative.
(b) There is a neutral element for $\perp$.

3. Let S be a nonempty set and let $\perp$ be a binary operation on S (assume its domain of definition is S^2). Express the statements below mathematically, without the negation operation.

(a) $\perp$ is not commutative.
(b) There is no neutral element for $\perp$.

4. Let S be a nonempty set, let $\perp$ be a binary operation on S (assume its domain of definition is S^2), and let n be a neutral element for $\perp$.

(a) Express mathematically that n is the *only* neutral element for $\perp$.
(b) Let x and y be two elements of S. We say that y in an ***inverse*** of x with respect to $\perp$ iff $x \perp y = y \perp x = n$. Express mathematically that every element of S has an inverse.

PART D.

1.

For ***(a)*** and ***(b)***, you may assume that the domain of each predicate is a nonempty finite set (i.e., a nonempty set with a finite number of elements).

(a) Show that: $\forall x, (P(x) \wedge Q(x)) \equiv (\forall x, P(x)) \wedge (\forall x, Q(x))$
(b) Show that: $\exists x, (P(x) \vee Q(x)) \equiv (\exists x, P(x)) \vee (\exists x, Q(x))$.
(c) Show that: $\forall x, (P(x) \vee Q(x)) \not\equiv (\forall x, P(x)) \vee (\forall x, Q(x))$
(d) Show that: $\exists x, (P(x) \wedge Q(x)) \not\equiv (\exists x, P(x)) \wedge (\exists x, Q(x))$.

2.

For ***(a)*** and ***(b)***, you may assume that the domain of P is the Cartesian product $X \times Y$, where X and Y are nonempty finite sets.

(a) Show that: $\forall x, \forall y, P(x,y) \equiv \forall y, \forall x, P(x,y)$
(b) Show that: $\exists x, \exists y, P(x,y) \equiv \exists y, \exists x, P(x,y)$.
(c) Show that: $\forall x, \exists y, P(x,y) \not\equiv \exists y, \forall x, P(x,y)$.