CIS1910 Discrete Structures in Computing (I)
Winter 2019, Lab 7 Notes

Here are recommended practice exercises on rules of inference and equation solving. Many have been covered in Lab 7.

PART A.

Note that $\neg$ is given the highest precedence.

1.
(a) Using truth tables, show that $p\rightarrow q \equiv \neg p \vee q$.
(b) Using truth tables, show that $[p\wedge(p\rightarrow q)]\rightarrow q$ is a tautology.
(c) Using the fundamental laws, show that $[\neg q\wedge(p\rightarrow q)]\rightarrow\neg p$ is a tautology.

2. The two tautologies above are of the form $(part_1\wedge part_2)\rightarrow part_3$ (for the first tautology, $part_1$=p, $part_2$=$p\rightarrow q$, and $part_3$=q; for the second tautology, $part_1$=$\neg q$, $part_2$=$p\rightarrow q$, and $part_3$=$\neg p$). They are, therefore, rules of inference:

$$\begin{array}{rl} & p \\ & p\rightarrow q \\ \hline \therefore & q \end{array} \qquad\qquad \begin{array}{rl} & \neg q \\ & p\rightarrow q \\ \hline \therefore & \neg p \end{array}$$

These rules are called ***modus ponens*** and ***modus tollens***, respectively. The modus ponens rule tells us that if p and q are two propositions such that p and $p\rightarrow q$ are true, then q must be true. The modus tollens rule tells us that if $\neg q$ and $p\rightarrow q$ are true, then $\neg p$ must be true. See slide 4.8 for other common rules of inference.

(a) Examples 6 and 7 from Section 1.6 of the textbook
(b) Exercises 1, 3, 5 and 31 from Section 1.6 of the textbook

PART B.

1. ***(a)*** Consider the propositional expressions below. For each one, use the fundamental laws to show that it is a tautology, rewrite it as a rule of inference, and explain what this rule tells us.

$[(p\rightarrow q)\wedge(q\rightarrow r)]\rightarrow(p\rightarrow r)$
$[(p\leftrightarrow q)\wedge(q\rightarrow r)]\rightarrow(p\rightarrow r)$
$[(p\rightarrow q)\wedge(q\leftrightarrow r)]\rightarrow(p\rightarrow r)$
$[(p\leftrightarrow q)\wedge(q\leftrightarrow r)]\rightarrow(p\leftrightarrow r)$

(b) Consider the propositional expressions below. Show that none of them is a tautology.

$[(p\rightarrow q)\wedge(q\rightarrow r)]\rightarrow(p\leftrightarrow r)$
$[(p\leftrightarrow q)\wedge(q\rightarrow r)]\rightarrow(p\leftrightarrow r)$
$[(p\rightarrow q)\wedge(q\leftrightarrow r)]\rightarrow(p\leftrightarrow r)$

2. Using "if… then…" properties (see Lab 1, Part B, Questions 1.1 and 1.2), solve over $\mathbb{R}$ the following equation in x:

$$\sqrt{4x+3}=\sqrt{7x+9}$$

(a) Give the most detailed answer you can: show each step; explain how you use each "if… then…" property; explain the role of the hypothetical syllogism (see slide 4.8) in solving the equation; explain how the solution set can be determined without making the fallacy of affirming the consequent (slide 4.11).

(b) Rewrite the answer above, so as to make it as concise as possible: show each step, but do not provide any justification; do not make any explicit reference to "if… then…" properties or rules of inference; where appropriate, use symbols like $\rightarrow$, $\wedge$, $\vee$ instead of words like if, then, and, or; finally, use $\begin{array}{l} p \\ \rightarrow q \\ \rightarrow r \end{array}$ as short for $\begin{array}{l} p\rightarrow q \\ q\rightarrow r. \end{array}$

3. Same as **2.**, but using "iff" properties instead (see Lab 1, Part B, Question 1.3).

(a) Give the most detailed answer you can: show each step, and explain how you use each "iff" property and rule of inference.

(b) Rewrite the answer above, so as to make it as concise as possible: show each step, but do not provide any justification; do not make any explicit reference to "iff" properties or rules of inference; where appropriate, use symbols like $\leftrightarrow$, $\wedge$, $\vee$ instead of words like iff, and, or; finally, use $\begin{array}{l} p \\ \leftrightarrow q \\ \leftrightarrow r \end{array}$ as short for $\begin{array}{l} p\leftrightarrow q \\ q\leftrightarrow r. \end{array}$