UNIVERSITY of GUELPH

CIS1910 Discrete Structures in Computing (I)
Winter 2019, Lab 9 Notes

Here are recommended practice exercises on proofs by induction.
Many of these exercises have been covered in Lab 9.

Note that the examples and exercises listed in blue come from the following textbook:
"Discrete Mathematics and Its Applications," by Rosen, Mc Graw Hill, 7th Edition
Be aware that there are some discrepancies between the lecture notes and this textbook:
the definitions and notations (and hence the proofs) are not always exactly the same.

For a proof by induction (whether it is strong induction or not), you should follow the structure given in class and summarized below. You are strongly advised to take a close look at the solutions to Assignment 3, Sections E and F.

PREDICATE
Start your proof by defining the predicate:
"Let P be the unary predicate whose domain is ... and such that P(n) is the statement: ..."

BASIS STEP
Then, you have to prove that P(1) (if 1 is the smallest element of the domain), or P(−2) (if −2 is the smallest element), or P(3), etc., depending on the exercise, is true.

INDUCTIVE STEP
Continue with the inductive step: "Let n be an arbitrary element of the domain. Assume P(n) is true. Let us show that P(n+1) is true." This step should obviously conclude with the sentence: "We have shown that P(n+1) is true."

CONCLUSION
The last sentence of your proof should be:
"By induction, we can conclude that: $\forall n, P(n)$"

PART A

1. Examples 2, 5, 8, 3 and 6 from Section 5.1 of the textbook.

2. Exercises 33, 3, 21, 31, 5, 23, 9, 11 and 15 from Section 5.1.

PART B

1. Example 3 from Section 5.2 of the textbook.

NOTE:

You will not be able to prove P(n+1) assuming P(n)
if you define the predicate P such that P(n) is the statement

"If the two piles contain n matches initially, the second player can guarantee a win."

However, you will be able to prove P(n+1) assuming P(n)
if you define P such that P(n) is the statement

*"If the two piles contain n matches **or less** initially, the second player can guarantee a win."*

P(n) just above can be reformulated as follows: *"For any integer k of the domain less than or equal to n, if the two piles contain k matches initially, then the second player can guarantee a win."* In other words, it is of the form ∀k≤n, Q(k). Here, Q(k) is the statement *"If the two piles contain k matches initially, then the second player can guarantee a win."* When P(n) is of that form, we talk about ***strong induction***. Therefore, the only difference between a proof by simple, ordinary induction and a proof by strong induction lies in how the statement P(n) is chosen.

2. Example 4 and Exercise 7 from Section 5.2.