UNIVERSITY of GUELPH

CIS1910, Midterm Exam, W19

- Last Name ___________ **KEY**
- First Name ___________ **KEY**
- Student ID ___________ **KEY**

There are 55 questions, including 5 BONUS questions.
23 of the questions are multiple-choice questions,
and 32 are short answer questions.
Each question is worth 1 point.

Part A: MULTIPLE-CHOICE QUESTIONS

Your answers must be recorded on the Scantron sheet.

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MIDTERM W19 – Part A

QUESTION 1

Consider the following four sentences:

(i) Can I say that 7×3+1=15?
(ii) 7×3+1=15 might be true; I'm not sure.
(iii) It is true to say that it is false to say that 7×3+1=15.
(iv) Consider a real number z."

How many of these sentences can you safely write in your assignment paper?

A. 0
B. 1
C. 2
D. 3
E. 4

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MIDTERM W19 – Part A

QUESTION 2

Consider the following four statements:

(i) 1 is the first term of the tuple.
(ii) 1 is the first element of the tuple.
(iii) 1 is the first term of the set.
(iv) 1 is the first element of the set.

How many of these statements may be correct, depending on the tuple or set?

A. 0
B. 1
C. 2
D. 3
E. 4

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MIDTERM W19 – Part A

QUESTION 3

Consider the following four statements:

(i) $\forall u, \forall v, P(u,v) \equiv \forall v, \forall u, P(u,v)$
(ii) $\forall u, \exists v, P(u,v) \equiv \exists v, \forall u, P(u,v)$
(iii) $\exists u, \forall v, P(u,v) \equiv \forall v, \exists u, P(u,v)$
(iv) $\exists u, \exists v, P(u,v) \equiv \exists v, \exists u, P(u,v)$

How many of these statements are correct?

A. 0
B. 1
C. 2
D. 3
E. 4

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MIDTERM W19 – Part A

QUESTION 4

Let P(u,v) be the statement: "u+v=u−v"
Assume the universe of both variables is $\mathbb{Z}$.
Now, consider the following four statements:

(i) $\forall u, \forall v, P(u,v)$
(ii) $\forall u, \exists v, P(u,v)$
(iii) $\exists u, \forall v, P(u,v)$
(iv) $\exists u, \exists v, P(u,v)$

How many of these statements are correct?

A. 0
B. 1
C. 2
D. 3
E. 4

MIDTERM W19 – Part A

QUESTION 5

Consider the following four statements:

(i) The length of the string zzyzx is 3.
(ii) The lengths of the strings zzyzxλ and zzyzx are the same.
(iii) The string zzyzx is an element of $\{x,y,z\}^5$.
(iv) The string zzyzx is the concatenation of the strings zz and yzx.

How many of these statements are correct?

A. 0
B. 1
C. 2
D. 3
E. 4

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MIDTERM W19 – Part A

QUESTION 6

Let x, y and z be three integers.
Consider the following four statements:

(i) x divides y iff:
$\exists m \in \mathbb{Z}, y = mx$
(ii) y is a multiple of x iff:
$\exists m \in \mathbb{Z}, y = mx$
(iii) x is a factor of y iff:
$\exists m \in \mathbb{Z}, y = mx$
(iv) z is a linear combination of x and y iff:
$\exists m \in \mathbb{Z}, \exists n \in \mathbb{Z}, z = mx + ny$

How many of these statements are correct?

A. 0
B. 1
C. 2
D. 3
E. 4

QUESTION 7

Consider the following four statements:

(i) There is a set S such that the set of all the subsets of S is {}.
(ii) There is a set S such that the set of all the subsets of S is {{}}.
(iii) There is a set S such that the set of all the subsets of S is $\{\emptyset,\{1\},\{2\},\{3\}\}$.
(iv) There is a set S such that the set of all the subsets of S contains exactly 64 elements.

How many of these statements are correct?

A. 0
B. 1
C. 2
D. 3
E. 4

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MIDTERM W19 – Part A

QUESTION 8

Consider the function $f : \mathbb{Z}^+ \to [0,5[$
$x \mapsto x^2+1$

(i) 0 has a preimage under f.
(ii) 1 has a preimage under f.
(iii) 2 has a preimage under f.
(iv) 5 has a preimage under f.

How many of the four statements above are correct?

A. 0
B. 1
C. 2
D. 3
E. 4

QUESTION 9

Let S be the solution set of some equation.
Assume that x is a solution of that equation iff x=1 or x=2.
Consider the four statements below:

(i) S = {}
(ii) S = {1}
(iii) S = {1,2}
(iv) S = {1,2,3}

How many of these statements may be correct, depending on the equation?

A. 0
B. 1
C. 2
D. 3
E. 4

QUESTION 10

Consider the subtraction, −, of real numbers.

(i) It is idempotent.
(ii) It is commutative.
(iii) It is associative.
(iv) There is a neutral element for it.
(v) There is an absorbing element for it.

How many of these five statements are correct?

A. 0
B. 1
C. 2
D. 3
E. 4

MIDTERM W19 – Part A

QUESTION 11

Let S be a set,
let − be a unary operation on S,
and let + and x be two binary operations on S.
For any elements u, v and w of S, we have:

(i) $(u+v)+w = u+(v+w)$
(ii) $(u+v)xw \neq u+(vxw)$
(iii) $-(u+v) = (-u)+(-v)$

How many of the three statements above are necessarily correct?

A. 0
B. 1
C. 2
D. 3

MIDTERM W19 – Part A

QUESTION 12

In this question, $(B,+,\cdot,\overline{\ \ })$ is a Boolean algebra.
The zero element is denoted by 0.

Consider the following statements:,

(i) $\forall(u,v,w)\in B^3$, $v=w \rightarrow u+v=u+w$
(ii) $\forall(u,v,w)\in B^3$, $u+v=u+w \rightarrow v=w$
(iii) $\forall(u,v,w)\in B^3$, $u+v=w \rightarrow u=w-v$
(iv) $\forall(u,v)\in B^2$, $u+v=u \rightarrow v=0$

How many of these four statements are correct?

A. 0
B. 1
C. 2
D. 3
E. 4

QUESTION 13

In this question, $(B,+,\cdot,\overline{\ })$ is a Boolean algebra.

Consider the following Boolean expressions:

(i) $x+\bar{x}+y$
(ii) $\bar{x}+\bar{y}+\bar{z}$
(iii) $x\cdot\bar{x}\cdot y$
(iv) $\bar{x}\cdot\bar{y}\cdot\bar{z}$

How many of these expressions are minterms of degree 3?

A. 0
B. 1
C. 2
D. 3
E. 4

QUESTION 14

Consider the following statements:

(i) ¬ has higher precedence than ∨
(ii) ∨ has higher precedence than ∧
(iii) ∧ has higher precedence than ↔
(iv) ↔ has higher precedence than →

How many of these four statements are correct?

A. 0
B. 1
C. 2
D. 3
E. 4

MIDTERM W19 – Part A

QUESTION 15

Consider $P : \mathbb{R} \to \mathscr{P}$
$u \mapsto P(u)$ where $P(u)$ is the statement "$|u|>u$".

Consider the propositions below:

(i) $\exists u \in \mathbb{R}, P(u)$
(ii) $\exists u \in \mathbb{R}^+, P(u)$
(iii) $\exists u \in \{\}, P(u)$

How many of these three propositions are true?

A. 0
B. 1
C. 2
D. 3

MIDTERM W19 – Part A

QUESTION 16

Consider the following statements:

(i) $\exists u, (P(u) \lor Q(u)) \equiv (\exists u, P(u)) \lor (\exists u, Q(u))$
(ii) $\exists u, (P(u) \land Q(u)) \equiv (\exists u, P(u)) \land (\exists u, Q(u))$

A. The only correct statement is (i)
B. The only correct statement is (ii)
C. Both statements are correct
D. None of these statements is correct

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MIDTERM W19 – Part A

QUESTION 17

Consider the following expressions:

(i) $\exists u, [(\forall v, P(u,v)) \lor Q(v)]$
(ii) $\exists u, [(\forall u, P(u,u)) \lor Q(u)]$

A. The only correct predicate expression is (i)
B. The only correct predicate expression is (ii)
C. Both expressions are correct predicate expressions
D. None of them are correct predicate expressions

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MIDTERM W19 – Part A

QUESTION 18

Consider the Boolean algebra $(\{0,1\},+,\cdot,^{-})$.
Let | be the Boolean operation defined by the following table:

x	y	x | y
0	0	1
0	1	1
1	0	1
1	1	0

Now, consider the four statements below:

(i) | is idempotent
(ii) | is commutative
(iii) There is a neutral element for |
(iv) There is an absorbing element for |

How many of these statements are correct?

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

MIDTERM W19 – Part A

QUESTION 19

Consider a Boolean algebra $(B,+,\cdot,^{-})$.
John has come up with a two-page proof for PROPOSITION 1.

> *PROPOSITION 1*
> *For any elements x and y of B we have: <some equality>*

Then, using the duality principle, he has derived PROPOSITION 2 from PROPOSITION 1:

> *PROPOSITION 2*
> *For any elements x and y of B we have: <some other equality>*

Which one of the statements below is the most appropriate?

A. PROPOSITION 1 is an axiom
B. PROPOSITION 2 is a theorem
C. PROPOSITION 1 is a lemma for PROPOSITION 2
D. PROPOSITION 2 is a corollary of PROPOSITION 1

MIDTERM W19 – Part A

QUESTION 20

Consider a function f. Let A be its domain and B its codomain.
Consider the following four statements:

(i) An element of A may have no image under f.
(ii) An element of A may have exactly two images under f.
(iii) An element of B may have no preimage under f.
(iv) An element of B may have exactly two preimages under f.

How many of these statements are true?

A. 0
B. 1
C. 2
D. 3
E. 4

MIDTERM W19 – Part A

QUESTION 21

If the rightmost digit in the base b expansion of $(2417)_b$ x $(95)_b$ is 5 then b might be:

(i) 10
(ii) 20
(iii) 30
(iv) 40

How many of these statements are true?

A. 0
B. 1
C. 2
D. 3
E. 4

MIDTERM W19 – Part A

QUESTION 22

Consider the following four propositions:

(i) $\forall x \in \mathbb{R}, \sqrt{x^2} = x$
(ii) $\forall x \in \mathbb{R}^+, \sqrt{x}^2 = x$
(iii) $\exists x \in \mathbb{R}, \sqrt{x^2} = x$
(iv) $\exists x \in \mathbb{R}^+, \sqrt{x}^2 = x$

How many of these propositions are true?

A. 0
B. 1
C. 2
D. 3
E. 4

QUESTION 23

Which one of these statements is correct?

A. There exists a set S such that $S^2xS = S^3$,
and there exists a set S such that $S^2xS \neq S^3$.

B. Whatever the set S, we have $S^2xS = S^3$.

C. Whatever the set S, we have $S^2xS \neq S^3$.

Part B: SHORT-ANSWER QUESTIONS

Your answers must be recorded on this exam booklet.

MIDTERM W19 – Part B

Consider the four statements below.
Which ones are true? Which ones are false?
Circle the appropriate responses (T or F).

(a)	0..0 = (0,0)	T	**F**
(b)	0..0 = {0,0}	**T**	F
(c)	0..0 = [0,0]	**T**	F
(d)	0..0 = [0,0[	T	**F**

MIDTERM W19 – Part B

Consider the four statements below.
Which ones are true? Which ones are false?
Circle the appropriate responses (T or F).

(a)	$-\infty..+\infty = \mathbb{R}$	T	**F**
(b)	$1..+\infty = \mathbb{N}$	T	**F**
(c)	$1..+\infty = \mathbb{Z}^+$	**T**	F
(d)	1..2 = 2..1	T	**F**

MIDTERM W19 – Part B

Consider the four propositions below.
Which ones are true? Which ones are false?
Circle the appropriate responses (T or F).

(a)	$\forall(x,y)\in\mathbb{R}^2$, $1/x = y \leftrightarrow x = 1/y$	**T**	F
(b)	$\forall(x,y)\in\mathbb{R}^2$, $\sqrt{x} = y \leftrightarrow x = y^2$	T	**F**
(c)	$\forall(x,y)\in\mathbb{R}^2$, $\lvert x\rvert = \lvert y\rvert \leftrightarrow x = y$	T	**F**
(d)	$\forall(x,y,z)\in\mathbb{R}^3$, $xz = yz \leftrightarrow x = y$	T	**F**

MIDTERM W19 – Part B

Consider the four propositions below.
Which ones are true? Which ones are false?
Circle the appropriate responses (T or F).

(a)	$13 = 3x2^2 + 0x2^1 + 1x2^0$	**T**	F
(b)	$13 = (301)_2$	T	**F**
(c)	$13 = 1x3^2 + 1x3^1 + 1x3^0$	**T**	F
(d)	$13 = (111)_3$	**T**	F

Let + and x be two binary operations on a set S.
We say that + is **distributive** over x iff:

$\forall(u,v,w)\in S^3$, u+(vxw)=(u+v)x(u+w) ∧ (vxw)+u=(v+u)x(w+u)

Let + be a binary operation on a set S, and let n be an element of S.
We say that n a **neutral element** for + iff:

$\forall u\in S$, u+n=n+u=u

Consider the Boolean algebra $(\{0,1\},+,\cdot,^{-})$.
Consider the Boolean function *G* defined by the table below:

x	*y*	*z*	*G(x,y,z)*
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

What is the product-of-sums expansion of *G*?

$(x+\overline{y}+z)\cdot(\overline{x}+\overline{y}+z)$

Consider a Boolean algebra $(B,+,\cdot,^{-})$.
Let ↓ be the Boolean operation defined by: $x \downarrow y = \overline{x+y}$
Show that for any elements x and y of B we have:
$x+y=(x\downarrow y)\downarrow(x\downarrow y)$
Justify each step.

$(x\downarrow y)\downarrow(x\downarrow y)$	$= (\overline{x+y})\downarrow(\overline{x+y})$	**definition of ↓**
	$= \overline{(\overline{x+y})+(\overline{x+y})}$	**definition of ↓**
	$= (\overline{\overline{x+y}})\cdot(\overline{\overline{x+y}})$	**De Morgan's laws**
	$= (x+y)\cdot(x+y)$	**double complement laws**
	$= x+y$	**idempotent laws**

Consider a Boolean algebra $(B,+,\cdot,\overline{\ })$.
Prove that for any elements x and y of B we have: $x+x\cdot y = x$
Justify each step.

$x+x\cdot y = x\cdot 1+x\cdot y$	**identity laws**
$= x\cdot(1+y)$	**distributive laws**
$= x\cdot 1$	**domination laws**
$= x$	**identity laws**

Consider a Boolean algebra $(B,+,\cdot,\overline{\ })$.
We can show that for any elements x and y of B we have:

$$x+x\cdot y = x$$

Therefore, according to the duality principle, we also have:

$$x\cdot(x+y) = x$$

MIDTERM W19 – Part B 51

Complete and make these propositions true.

(1) $\forall(a,b)\in\mathbb{R}^2$, (a=b ___**$\wedge$ a≠0**___) $\rightarrow$ 1/a = 1/b

(2) $\forall(a,b,c)\in\mathbb{R}^3$, ac=bc $\rightarrow$ (a=b ___**$\vee$ c=0**___)

MIDTERM W19 – Part B 52

Complete and make these propositions true.

(1) $\forall(a,b)\in\mathbb{R}^2$, $a^2=b^2$ $\leftrightarrow$ (a=b ___**$\vee$ a=−b**___)

(2) $\forall(a,b)\in\mathbb{R}^2$, $\sqrt{a}=\sqrt{b}$ $\leftrightarrow$ (a=b ___**$\wedge$ a≥0**___)

MIDTERM W19 – Part B 53

Complete and make this proposition true.

$\forall(a,b)\in\mathbb{R}^2, \sqrt{a} \geq b \leftrightarrow (a \geq b^2 \land$ **b≥0** $) \lor (a \geq 0 \land$ **b<0** $)$

MIDTERM W19 – Part B 54

Consider the function ({x,y},{0,1},{(x,0)}).

Its domain is **{x,y}**

Its codomain is **{0,1}**

Its domain of definition is **{x}**

Its range is **{0}**

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MIDTERM W19 – Part B 55

Consider the relation ({x,y},{0,1},{(x,0),(x,1)}).

(a) Draw its arrow diagram:

arrow diagram

(b) Is it a function (yes/no)? ______**no**______

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MIDTERM W19 – Part B 56

Find the base 10 expansion of $(21331)_4$.

×4+ ➔

2	**1**	**3**	**3**	**1**
	9	39	159	**637**

$(21331)_4$ = **637**

Find the base 5 expansion of 534.

←

0	4	21	106	534	div 5
	4	1	1	4	mod 5

534 = $(4114)_5$

Find the base 4 expansion of $(1A7)_{16}$.

$(1A7)_{16} = (1\ 22\ 13)_4 = (12213)_4$

Find the base 16 expansion of $(11110)_2$.

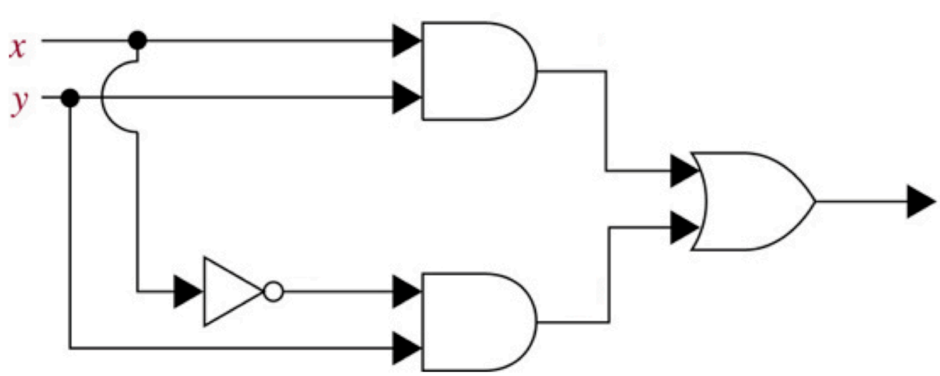
$(11110)_2 = (1\ 1110)_2 = (1E)_{16}$

Here is how to calculate the base 8 expansion of the sum:
$(751)_8 + (743)_8$

$$\begin{array}{rcccc} & ^{1} & ^{1}7 & 5 & 1 \\ + & & 7 & 4 & 3 \\ = & \mathbf{1} & \mathbf{7} & \mathbf{1} & \mathbf{4} \end{array}$$

What is the output to the circuit below?

$x \cdot y + \overline{x} \cdot y$



MIDTERM W19 – Part B 61

Consider the Boolean algebra ({0,1},+,·,‾)
and a Boolean function F whose sum-of-products expansion is:

$$x \cdot \overline{y} \cdot z + x \cdot y \cdot z$$

Construct the circuit that produces the output F(x,y,z).

circuit

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MIDTERM W19 – Part B 62

Consider the Boolean algebra ({0,1},+,·,‾)
and a Boolean function F whose sum-of-products expansion is:

$$x \cdot \overline{y} \cdot z + x \cdot y \cdot z$$

Minimize F. Justify each step.

F(x,y,z) =	$\mathbf{x \cdot \overline{y} \cdot z + x \cdot y \cdot z}$	**definition of F**
	$= (x \cdot \overline{y} + x \cdot y) \cdot z$	**distributive laws**
	$= x \cdot (\overline{y} + y) \cdot z$	**distributive laws**
	$= x \cdot 1 \cdot z$	**complement laws**
	$= x \cdot z$	**identity laws**

MIDTERM W19 – Part B

Assume E(x,y) is the statement: "x has sent an email to y."
Translate each of the following into English.
The sentences should sound as natural as possible.

$\forall x, \forall y, E(x,y)$

Everybody has sent an email to everybody.

$\forall x, \exists y, E(x,y)$

Everybody has sent an email to somebody.

$\exists x, \forall y, E(x,y)$

Somebody has sent an email to everybody.

$\exists x, \exists y, E(x,y)$

Somebody has sent an email to somebody.

MIDTERM W19 – Part B

Assume E(x,y) is the statement: "x has sent an email to y"
Translate each of the following into English.
The sentences should sound as natural as possible.

$\forall x, \forall y, x=y \rightarrow E(x,y)$

Everybody has sent an email to themselves.

$\forall x, \forall y, E(x,y) \rightarrow x=y$

Nobody has sent any email, except maybe to themselves.

MIDTERM W19 – Part B 65

Assume E(x,y) is the statement: "x has sent an email to y"
Translate each of the following into English.
The sentences should sound as natural as possible.

$\forall x, \forall y, x \neq y \rightarrow E(x,y)$

Everybody has sent an email to everybody,

except, maybe, to themselves.

$\forall x, \forall y, E(x,y) \rightarrow x \neq y$

Nobody has sent an email to themselves.

MIDTERM W19 – Part B 66

Assume P(x) is the statement: "x got an A+"
Write a predicate expression that translates into the following sentence:
"At most one student got an A+."

$\forall x, \forall y, P(x) \land P(y) \rightarrow x=y$

MIDTERM W19 – Part B 67

Assume P(x) is the statement: “x got an A+”
Translate the following into English.
The sentence should sound as natural as possible.

$\exists x, (P(x) \land \forall y, (P(y) \rightarrow y=x))$

Only one student got an A+

MIDTERM W19 – Part B 68

Translate each of the following statements into a propositional expression involving exactly two propositions, p and q. In each case, specify what p and q are.

“It is necessary to wash the boss’ car to get promoted.”

$p \rightarrow q$ with p=“you get promoted” and q=“you wash the boss’ car”

“A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.”

$p \rightarrow q$ with p=“you bought the computer less than a year ago”

and q=“the warranty is good”

Show that the following propositional expression is a rule of inference:
$[(p \vee q) \wedge \neg p] \rightarrow q$

truth table

What is the standard notation for this rule of inference?

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline \therefore q \end{array}$$

{1,2}
is a pair set that does not contain 0 and does not contain {0} either.

{0,1}
is a pair set that contains 0 but does not contain {0}.

{{0},{1}}
is a pair set that does not contain 0 but contains {0}.

{0,{0}}
is a pair set contains both 0 and {0}.

MIDTERM W19 – Part B

Let A={0} and B={1,2}.

AxB = **{(0,1),(0,2)}**

BxA = **{(1,0),(2,0)}**

This shows that the Cartesian product is

not commutative

MIDTERM W19 – Part B

To answer this question, please refer to the next two pages.

Complete each line with the name of the appropriate rule of inference:

1. $\forall x, (r(x) \rightarrow t(x))$	Premise	
2. $r(\text{Linda}) \rightarrow t(\text{Linda})$	**universal instantiation**	using 1.
3. r(Linda)	Premise	
4. t(Linda)	**modus ponens**	using 2. and 3.
5. c(Linda)	Premise	
6. $c(\text{Linda}) \wedge t(\text{Linda})$	**conjunction**	using 4. and 5.
7. $\exists x, (c(x) \wedge t(x))$	**existential generalization**	using 6.