Describing Topological Relationships in Words: Refinements

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***Abstract*—In earlier work, we introduced a method for generating linguistic descriptions of the topological relationships between two-dimensional objects. The input to the system is a pair of rasterized objects and the output is a set of propositions about their spatial relationships expressed in natural language. The method relies on finding one or two Allen relations that best describe the relationships along a direction of major object interaction. In this paper, we address some of the issues related to the use of Allen relations for describing two-dimensional object configurations, and we propose two extensions in order to solve problems encountered in the original algorithm. Global subsethood-based information is used to suppress counter-intuitive descriptions and an ancillary method for generating alternative descriptions is introduced.**

I. Introduction

The modeling of spatial relationships has received a lot of attention in recent years due to rapid developments in the GIS community. Numerous methods have been proposed to represent the relationships between two-dimensional objects. The region connection calculus [11,12], for instance, or the n-intersection models of Egenhofer *et al*. [2,3,4], are simple and powerful representation models that have gained a wide following. Work in the modeling of topological relationships is often based on the extension of Allen's temporal relations [1] into the spatial domain. Allen relations form a set $\mathcal{A}$ = {<,*m*,*o*,s,*fi*,*d*,=,*di*,*si*,*f*,*oi*,*mi*,>} of thirteen mutually exclusive and collectively exhaustive relations that can hold between two segments on an oriented line (Fig. 1). Allen relations are of interest to the GIS community because they can be used to model human temporal and spatial reasoning more adequately than models based strictly on numerical specifications. In a recent publication [8], Matsakis and Nikitenko introduced the concept of Allen F-histograms. These histograms, which contain a wealth of information, were used in [9] to capture the essence of the topological relationships between 2D objects using natural language descriptions. Each linguistic description relies on the computation of the thirteen Allen F-histograms and attempts to extract the essential characteristics of the object configuration while leaving out superfluous and possibly overwhelming detail. It consists of three components: a topological component that summarizes the primary Allen relations existing along a direction of major object interaction, a self-assessment component which reflects the complexity of the configuration, and, whenever relevant, one or more directional estimate(s) of where the primary Allen relations are most prominent.

Relying on the thirteen Allen relations only, the system presented in [9] is usually able to synthetically describe the scene in a very reasonable way (Fig. 2). Occasionally, however, it produces counter-intuitive descriptions (Fig. 3). In Section III.A, we propose a method to suppress these counter-intuitive descriptions. When the system is unable to describe the scene at all, it simply acknowledges it. In Section III.B, however, we propose an ancillary method for producing alternative descriptions based on subsethood information. Results are discussed in Section IV, and some final thoughts are given as conclusion in Section V. We begin with a brief presentation of the original method used to select the most appropriate Allen relations for the linguistic description.

II. Original method

A. Allen F-Histograms

The notion of the F-histogram was introduced in [6]. F-histograms include force histograms and Allen histograms [8]. We focus here on Allen histograms. Consider two objects A and B, and an Allen relation *r*. The histogram F_r^{AB} is one possible representation of the position of A (the *argument*) with regard to B (the *referent*). It is a numeric function. For

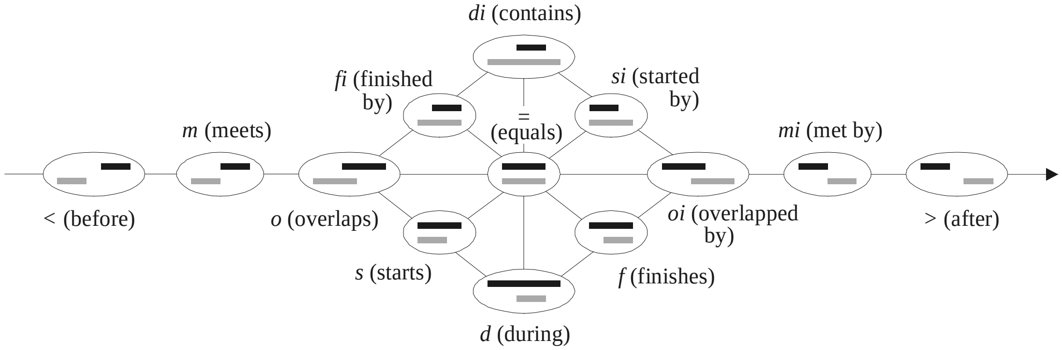

Fig. 1. Allen relations [1] between two segments on an oriented line. The black segment is the *referent*, the gray segment is the *argument*. Two relations r_1 and r_2 are linked if and only if they are conceptual neighbors [5], i.e., r_1 can be obtained directly from r_2 by moving or deforming the segments in a continuous way.

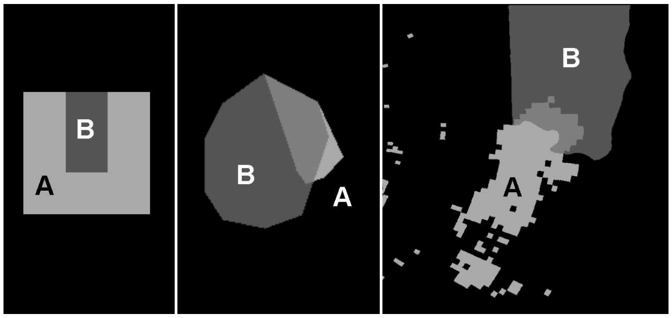

Fig. 2. The system presented in [9] usually produces reasonable descriptions. *Left*: "A perfectly meets B. The description is satisfactory. A meets B in multiple directions." *Middle*: "A starts and slightly overlaps B. The description is satisfactory. A starts B to the East-Northeast and overlaps B to the East." *Right*: "A is before and somewhat meets B. The description is rather satisfactory. A is before B to the Southwest and meets B to the South-Southwest."

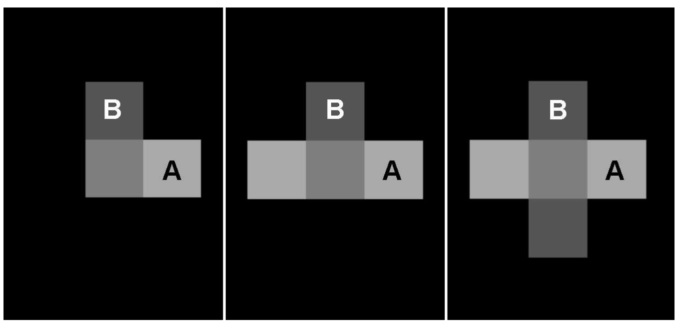

Fig. 3. The system presented in [9] occasionally produces counter-intuitive descriptions. *Left*: "A starts and is marginally equal to B. The description is rather satisfactory. A starts B to the South." *Middle*: "A contains and marginally starts B. The description is satisfactory. A starts B loosely to the South." *Right*: "A contains and marginally starts B. The description is rather satisfactory. A starts B in multiple directions."

any direction θ, the value $F_r^{AB}(\theta)$ is a weight attached to the proposition "A r B in direction θ." This weight is computed by subdividing A and B into pairs of *longitudinal sections* (Fig. 4). The handling of the pair (A,B) comes down to the handling of pairs of longitudinal sections, and the handling of each pair of longitudinal sections comes down to the handling of pairs of segments for which the Allen relation r can be assessed. Three aspects of the computation of F_r-histograms are of interest here.

First, the Allen relations are fuzzified. The idea is that if an object is moved slightly then the Allen F-histograms should change equally slightly. Let $\mathcal{A}$ be the set of all thirteen fuzzy Allen relations. Fuzzification is achieved such that any r in $\mathcal{A}$ is a continuous function onto [0,1]. Moreover, for any pair (I,J) of segments on an oriented line, $\Sigma_{r\in\mathcal{A}}\, r(\mathrm{I,J}) = 1$. For any pair (I,J) and any r_1 and r_2 in $\mathcal{A}$, if $r_1(\mathrm{I,J})\neq 0$ and $r_2(\mathrm{I,J})\neq 0$ then r_1 and r_2 are direct neighbors in the graph of Fig. 1. An illustrative example is presented in Fig. 5.

Second, the longitudinal sections are also fuzzified. The idea here is that if an object is deformed slightly then the Allen F-histograms should change equally slightly. Fuzzification is achieved such that the closer two segments of a crisp longitudinal section are, the more the space in between belongs to the fuzzified section. When sufficiently close, the two segments are almost seen as a single segment. An illustrative example is presented in Fig. 6.

Finally, the processing of a pair of longitudinal sections is achieved by processing their α-cuts and blending the $r(\mathrm{I,J})$ values appropriately. Moreover, the overall contribution $r(\mathrm{I,J})$ of each pair of segments is weighed such that $\Sigma_{r\in\mathcal{A}}\, F_r^{AB}(\theta)$ measures the object interaction in direction θ. Put simply, $\Sigma_{r\in\mathcal{A}}\, F_r^{AB}(\theta)$ is the total area of the regions of A and B that are facing each other in direction θ (Fig. 7). Again, a slight change in the object configuration results in a correspondingly slight change in the histograms. Continuity is satisfied and, hence, robustness is achieved.

All details pertaining to the computation of F_r-histograms are presented in [8]. A comprehensive summary appears in [13]. In order to describe, in natural language, the topological relationships between A and B, the system in [9] analyzes the thirteen histograms F_r^{AB} and selects a set of at most two Allen relations. The two relations do not semantically contradict each other and are the most representative along some direction of major object interaction. At this point, we turn our attention to the selection process.

B. *Coherent Sets of Allen Relations*

The description "A meets and slightly overlaps B" sounds coherent since it is easy to picture two such objects A and B. We say that $\{m,o\}$ is a *coherent set* of Allen relations. On the other hand, "A contains and is before B" goes against intuition. The relations involved in this description seem somewhat contradictory. $\{di,<\}$ is not a coherent set. The concept of coherent set was initially explored in [13] and a more detailed account appears in [9]. In this paper, the coherent sets are all the singletons (e.g., $\{<\}$, $\{m\}$, $\{o\}$) and all the pairs of direct neighbors in the graph of Fig. 1 (e.g., $\{<,m\}$, $\{m,o\}$, $\{o,s\}$). Any one of these coherent sets can be used to generate a linguistic description. In order to select the "best" set, we introduce the notion of *satisfactory indices*.

C. *Local Satisfactory Indices*

A local satisfactory index, $\sigma_c(\theta)$, measures the degree to which the Allen relations in the coherent set c satisfactorily represent the spatial relationships between A and B along direction θ. The term "local" reflects the fact that only direction θ is considered. $\sigma_c(\theta)$ belongs to the interval [0,1] and is 1 if and only if the relations in c are the only Allen relations present along θ. The simplest way to define $\sigma_c(\theta)$ is as follows:

$$\sigma_c(\theta) = \Sigma_{r\in c}\, v_r(\theta),$$
$$\text{where}\quad v_r(\theta) = F_r^{AB}(\theta) \,/\, \Sigma_{\rho\in\mathcal{A}}\, F_\rho^{AB}(\theta).$$

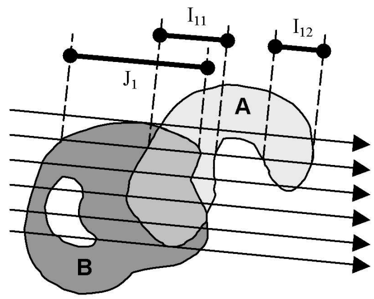

Fig. 4. For each direction, the plane is partitioned into a set of parallel lines. The intersection of a line and an object is a longitudinal section of the object. J_1, for instance, is a longitudinal section of B. It consists of one segment. I_1 is a longitudinal section of A and consists of two disjoint segments I_{11} and I_{12}.

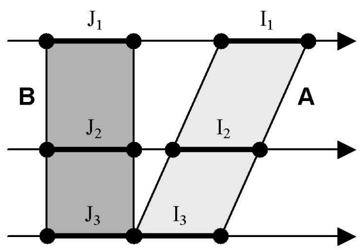

Fig. 5. Fuzzy Allen relations. Here, $>(I_1,J_1)$ is 1 and $mi(I_1,J_1)$ is 0; $>(I_2,J_2)$ and $mi(I_2,J_2)$ are greater than 0 and less than 1; $>(I_3,J_3)$ is 0 and $mi(I_3,J_3)$ is 1.

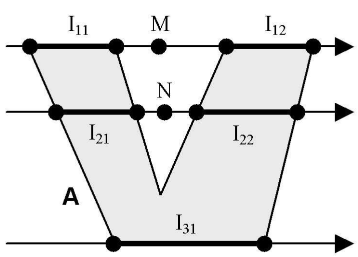

Fig. 6. Fuzzy longitudinal sections. Before fuzzification, the point M does not belong at all to $I_1=I_{11}\cup I_{12}$ and N does not belong at all to $I_2= I_{21}\cup I_{22}$. After fuzzification of I_1 and I_2, the point M belongs to I_1 to some extent, and N belongs to I_2 more than M belongs to I_1.

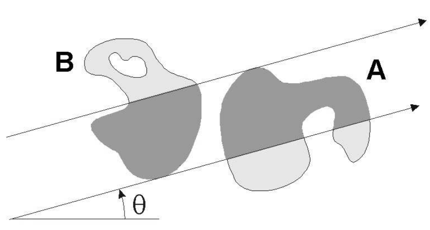

Fig. 7. The value $\Sigma_{r\in\mathcal{A}}\, F_r^{AB}(\theta)$ measures the object interaction in direction θ. In this example, it is the total area of the two dark gray regions.

A somewhat more sophisticated definition might be:

$$\sigma'_c(\theta) = \max\,\{0\ ,\ \sigma_c(\theta) - \Sigma_{r\in\mathcal{A}-c}\,(\delta_{rc}/6)\ v_r(\theta)\},$$

where δ_{rc} denotes a weighted average conceptual distance between the Allen relation r and the relations in c. The value δ_{rc} belongs to [0,6]. It is 0 when, e.g., r is < and c is {<}. It is 6 when, e.g., r is > and c is {<} (in the graph of Fig. 1, the length of the shortest path between > and < is 6). Every relation $r\in\mathcal{A}$–c that coexists with c along θ makes $\sigma'_c(\theta)$ decrease, and the higher its distance to c, the bigger the decrease. However, one might also want to take into account the *degree of interaction* between the objects along direction θ. Here is a third possible definition:

$$\sigma''_c(\theta) = \min\,\{\sigma'_c(\theta)\ ,\ i(\theta)\}\ ,$$

where $$i(\theta) = \Sigma_{r\in\mathcal{A}}\, F_r^{AB}(\theta)\ /\ \max_\varphi \Sigma_{r\in\mathcal{A}}\, F_r^{AB}(\varphi)\ .$$

$\sigma''_c(\theta)$ measures to what extent the relations in c dominate the topological relationships along θ *and* whether object interaction along θ is high. The differences between σ, σ' and σ'' are illustrated in Fig. 8. More about local satisfactory indices can be found in [9,13]. In particular, these papers discuss some issues related to directional inverses (like *m* and *mi*).

D. Global Satisfactory Indices

In [9], the global satisfactory index of a coherent set c is defined as:

$$s_c = \max_\theta \sigma''_c(\theta)\ .$$

It measures the degree to which the relations in c satisfactorily represent the spatial relationships along some direction. Here, the operator max can be interpreted as a fuzzy existential quantifier. The winning set of Allen relations for the linguistic description is a coherent set c_0 such that:

$$s_{c_0} = \max_c\,(s_c)\ .$$

The self-assessment component of the linguistic description is based on the global satisfactory index of the winning coherent set. If s_{c_0} is low, none of the coherent sets can be used to satisfactorily describe the spatial relationships. This typically occurs when there is a lot of ambiguity along the directions with high object interaction. The presence of contradictory relations along these directions drives s_{c_0} towards zero. If, on the other hand, there exists a coherent set whose relations satisfactorily represent the relationships along some direction with high object interaction, s_{c_0} tends towards 1 (its maximum possible value).

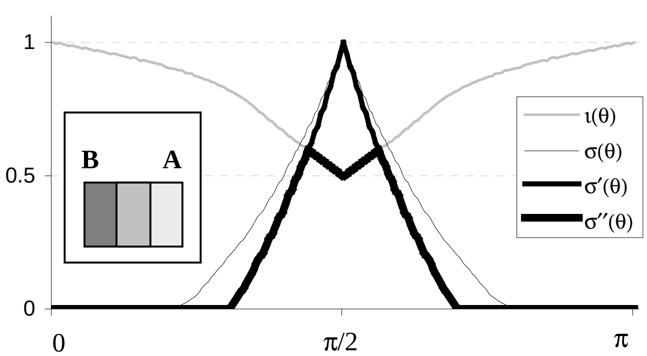

Fig. 8. Plots of different satisfactory indices associated with the coherent set {=} and the pair of overlapping objects shown in the inset on the left.

III. Proposed Extensions

A. Global Subsethood Information

In most cases, the system presented in [9] produces intuitive descriptions which satisfactorily capture the essence of the object configurations. In some cases, however, the system fails. The problem with the global satisfactory index as defined in Section II.D lies in the fact that, in the end, the focus is given to a single direction. Certainly, the algorithm could benefit from a more global view of the scene. Consider the objects in Fig. 3b. The argument object A contains the referent object B along the horizontal direction. To say that "A contains B," however, does not accurately describe the topological relationships in the 2D space. In this sense, the description generated by the system in [9] is counter-intuitive. In this section, we propose a method to suppress such invalid descriptions.

The idea is to make use of some ancillary information about the spatial relationships between the objects. We utilize here the notion of *subsethood*. The degrees of subsethood between the objects are calculated and incorporated in the computation of the satisfactory indices. For instance, the degree of subsethood of B in A can be seen as the maximum possible degree of truth of the proposition "A contains B" (i.e., "A *di* B"). Clearly, if B is not a subset of A, it cannot be contained by A and any description including the relation *contains* should be suppressed. Let us take a closer look at this idea. The degree of subsethood of B in A is often defined by $|B \cap A|/|B|$, i.e., the proportion of B which coincides with A. Intuitively, however, this definition is somewhat unsatisfying. For the objects in Fig. 3a, for instance, the degree of subsethood of B in A would be 0.5, while a human observer would likely say that B is not a subset of A at all. In this paper, we therefore prefer to define the subsethood of B in A as $\mu_{sub}(B,A)$, where μ_{sub} is the membership function shown in Fig. 9. In the end, and in the light of global subsethood information, the maximum degree of truth that can reasonably be attached to the proposition "A *r* B" is the value V_r as defined in Table I.

$V_=$ deserves a few words of explanation. In Table I, it is defined as $\max\{\mu_{sub}(A,B)\ ,\ \mu_{sub}(B,A)\}$. This may, at first, seem to go against the grain of reason. A common way to think of equality is to say that it occurs whenever A is a subset of B *and* B is a subset of A. In this light, $V_=$ would have been better defined as $\min\{\mu_{sub}(A,B)\ ,\ \mu_{sub}(B,A)\}$, where the min operator can be interpreted as the standard fuzzy conjunction. In this context, however, it is more convenient to treat equality as a special case of subsethood, in the sense that whenever subsethood exists, there may also be partial equality, regardless of whether A is a subset of B, B is a subset of A, or both.

The simplest way to incorporate global subsethood-based information into the existing algorithm is to modify the local satisfactory index:

$$\sigma'''_c(\theta) = \min\{\sigma''_c(\theta)\ ,\ \Sigma_{r\in c} \min(v_r(\theta)\ ,\ V_r)\}\ .$$

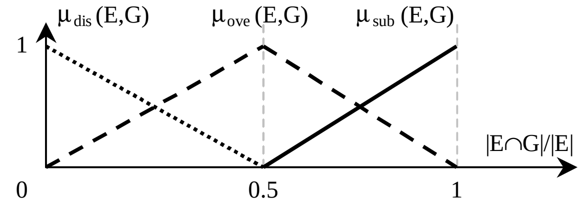

Fig. 9. Membership functions defining subsethood, overlap and disjointness.

TABLE I
CLIPPING VALUES USED IN THE COMPUTATION OF SATISFACTORY INDICES

Relation *r*	**Clipping Value V_r**
<,>,*m*,*mi*	$\min\{\mu_{dis}(A,B)\ ,\ \mu_{dis}(B,A)\}$
o,*oi*	$\min\{\mu_{ove}(A,B)\ ,\ \mu_{ove}(B,A)\}$
d,*s*,*f*	$\mu_{sub}(A,B)$
di,*si*,*fi*	$\mu_{sub}(B,A)$
=	$\max\{\mu_{sub}(A,B)\ ,\ \mu_{sub}(B,A)\}$

The local satisfactory index was upper bounded by $\Sigma_{r\in c} v_r(\theta)$, it is now upper bounded by $\Sigma_{r\in c} V_r$ also. If $v_r(\theta)$ exceeds V_r, its contribution to the satisfactory index is "clipped" to the maximum reasonable level defined by V_r. The value $\Sigma_{r\in c} \min(v_r(\theta),V_r)$ can be interpreted as a measure of agreement between the relations in the coherent set c along direction θ and the subsethood information. If the agreement is high, we can be sure that the relations in c not only represent well the topological relationships along θ, but also reflect the spatial relationships in the 2D space. The modified system then produces the same linguistic description as the original system. If the agreement is low, however, the local satisfactory index of c is quickly driven below the threshold required for an acceptable linguistic description [9]. Instead of generating counter-intuitive and potentially misleading statements, the modified system acknowledges its inability to describe the scene and outputs the following message: "The spatial relationships cannot be assessed." This is illustrated in Section IV.

B. Description by Parts

When the system is unable to describe the scene using only Allen relations, it informs the user and gives up (see Section III.A). We propose here an ancillary method for producing alternative descriptions based on subsethood information.

It can be shown that situations where no coherent set of Allen relations is good enough for a description occur only if there exists *partial coincidence* between the objects A and B. Consider the configurations (a), (b) and (c) in Fig. 3. In all of these examples, there is a conflict related to partial coincidence (partial mutual containment). In each case, part of the referent contains part of the argument and, inversely, part of the argument contains part of the referent. A is a partial subset of B and B is a partial subset of A. Even though the topological relationships in the 2D space are fairly straightforward, no coherent set of Allen relations can be used to satisfactorily describe these relationships.

Appealing once again to some basic elements of set theory, alternative descriptions can be generated for such configurations. The descriptions will tell the user which parts of the objects coincide, as in, for instance, "The southern part of B coincides with the central part of A" (Fig. 3b). More formally, they will take the following form:

"The <$which_B$> part of B coincides
with the <$which_A$> part of A."

To obtain $which_B$, we partition B into two disjoint subregions $B_1 = B-A$ and $B_2 = B \cap A$. The symbol B_1 denotes the part of B which is disjoint from A, and the symbol B_2 denotes the part of B which coincides with A. In order to obtain $which_B$, we assess the relative position of B_2 with respect to B_1. In our experiments, we have utilized histograms of constant forces [8,10] for this purpose. Force histograms are particular F-histograms which have been successfully applied in generating linguistic descriptions using the primitive directional relations "to the right of," "to the left of," "above," and "below" [7]. Whether harnessing such power for the task at hand is justified is a matter deserving further investigation. Perhaps simpler (and computationally cheaper) methods would suffice here, like the centroid or MBR-based methods.

Once the histogram of constant forces associated with the object pair (A,B) is computed, a primary direction is extracted from it, using the technique proposed in [7]. The primary direction is then converted to a linguistic value. If no satisfactory direction can be found, $which_B$ is set to "central," since this case occurs when the argument object lies in two (or more) opposing directions of the referent. $which_A$ is obtained using the same technique.

The "Description by Parts" module is activated whenever the system is unable to describe the scene using only Allen relations. This approach is still in early stages of development. It is presented here as a potential novel method of assessing the topological relationships between two objects, based strictly on the directional relationships of their sub-components. It is also one more interesting application of force histograms.

IV. Results

Relying on the Allen relations only, the system presented in [9] is usually able to synthetically describe the scene in a very reasonable way. This is illustrated by the configurations in Fig. 2. For these configurations, the original system [9] and the modified systems (Sections III.A and III.B) produce the exact same linguistic descriptions. Occasionally, however, the system in [9] produces counter-intuitive descriptions. Figure 3 shows three typical examples of problematic configurations. The original descriptions seem rather counter-intuitive, as they fail to fully capture the topological relationships in the 2D space. Figure 3a depicts an L-shaped configuration, but the system determines that the dominant relation is *s* (i.e., A *starts* B). This is true along the vertical direction $\theta=\pi/2$ (from South to North), but what of the horizontal direction $\theta=0$ (from West to East)? A is *started by* B. This fact is completely omitted from the description.

After applying the global subsethood-based information (Section III.A), it turns out that no pertinent Allen relation(s) can be selected for the description. In the light of subsethood information, the maximum degree of truth that can reasonably be attached to the proposition "A *r* B" is 1 if *r* is *o* or *oi* (i.e., $V_o=V_{oi}=1$) and 0 otherwise. This indicates that the only possible way to describe the scene would be to use the *overlaps* and *overlapped by* relations. These relations, however, do not occur to any significant degree along any direction. Hence, no set of Allen relations can be selected for the description. The system modified as in Section III.A acknowledges its inability to describe the scene and outputs the message "The spatial relationships cannot be assessed" (Table II). Going one step further and activating the Description by Parts module (Section III.B), we are able to obtain a reasonable linguistic description. Here, no attempt is made to use Allen relations. We merely try to describe which parts of the two objects coincide. Figures 3b and 3c depict similar configurations, but with T-shaped and X-shaped objects, respectively. Again, the original, counter-intuitive statements are suppressed and replaced by very reasonable alternative descriptions.

TABLE II
LINGUISTIC DESCRIPTIONS PRODUCED BY THE MODIFIED SYSTEMS

	System as in III.A	**System as in III.B**
Fig. 3a	The spatial relationships cannot be assessed.	The southern part of B coincides with the western part of A.
Fig. 3b	The spatial relationships cannot be assessed.	The southern part of B coincides with the central part of A.
Fig. 3c	The spatial relationships cannot be assessed.	The central part of B coincides with the central part of A.

V. Conclusions

The thirteen Allen F-histograms associated with a pair of 2D objects encapsulate a wealth of topological, directional, and metric information. The system presented in [9] makes this information more accessible. It uses the Allen F-histograms to capture, through natural language descriptions, the essence of the topological relationships between the objects. Although often successful, the system occasionally produces counter-intuitive descriptions. In this paper, we have presented a simple extension to the system in order to suppress these descriptions. As a result, the descriptive process may fail, but such failures are preferable to generating misleading descriptions. Moreover, we have shown that it is possible to produce very reasonable alternative descriptions by taking advantage of some global subsethood information. The results demonstrate the effectiveness of these extensions. In future work, we intend to develop a unified framework for generating linguistic descriptions of the spatial relationships between 2D objects. Furthermore, we intend to investigate some validation algorithms to assess the accuracy of the descriptions.

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